A Chi-Square Statistic for Checking Satisfactory Edge Matching of Maps and Diagrams that Depict Adjacent Areas

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Key words: Cadastre, quality control, edge matching, Hellenic Cadastre, maps.

ABSTRACT

This paper describes a statistical method that can be used to determine whether two or more maps produced by independent surveys and cover adjacent areas match each other satisfactorily. This situation often arises when adjacent areas are mapped by different surveyors or at different times, using different sets of geodetic control measurements. The method is based on comparing differences in the coordinates of points that are well identified on pairs of adjacent maps (or diagrams). These differences are used to calculate a statistic that is suitable to determine whether the joint observed differences in the coordinates could be attributed to random error or not. It is shown that the statistic follows the *Chi-square* distribution and, therefore, it is amenable to treatment by the standard statistical methods. The use of the method in practice is demonstrated through indicative examples from Hellenic Cadastre Project. Indeed, within the context of the Quality Control procedure of the Hellenic Cadastre Project, the 1:5000 scale orthophotomaps, which are produced by different contractors and are based on different sets of geodetic control computations, are checked for satisfactory edge matching. The results of the procedure are described and discussed for a case study at the island of Chios, Greece.

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1. INTRODUCTION

In mapping applications, it is often the case that maps of two or more adjacent areas must be matched to form a new map that depicts all those areas. Each of those maps may have been compiled independently and may have been based on different sets of survey measurements. Since those measurements contain errors, it is likely that, when matching takes place, maps or diagrams may not match each other exactly. This problem is of particular importance in topographic, photogrammetric, and cadastral applications, where the precision requirements of maps and diagrams are stricter than in other contexts. In the past, when maps existed only in analogue form, this problem was not so evident because of the inability of the human eye to distinguish discrepancies beyond a given level and because analogue maps were not processed further directly. Moreover, maps often were made covering complete map sheets and, therefore, this problem was arising only when adjacent map sheets had to be matched to form a unified continuum. With the advent of the digital systems, however, this problem becomes more evident because, in a digital environment, the capability to change scales is unlimited and, therefore, discrepancies that are not evident at one scale may become evident at another. More importantly, the topological connections among cartographic features that, in the context of reading analogue maps, are viewed instinctively by the human eye, cannot be made by default in a digital context. Instead, they must be made by explicit processing of the data in order to ensure that correct connections are made and stored in digital databases.

The problems in matching maps and diagrams of adjacent areas are of particular interest also in cases where mapping agencies outsource mapping projects to independent contractors and, at the end, must unify the deliverables. In this case, it must be determined whether the maps and diagrams made by one contractor match satisfactorily those made by the other. Any discrepancies that are observed should be attributed solely to the specified error of the underlying surveying or photogrammetric measurements and not to any other source (bias or systematic error). It must be noted that those errors are inherent and are present in all cases, independently of the methods, topographic or photogrammetric, that are used to make the maps.

The majority of the literature (e.g. FGDC 1998; Greenfeld 2001) is focused more on establishing that each map meets independently certain accuracy criteria than on testing directly whether two maps depicting adjacent areas match each other satisfactorily. Indeed, in traditional approaches, certain well-identified features depicted on the maps are selected and their coordinates, as measured on those maps, are contrasted with the corresponding coordinates obtained from other, higher accuracy, sources (e.g. field measurements). Typically, indices, such as the Root Mean Square Error (RMSE), are used to quantify accuracy and compare it with pre-set standards or benchmarks. Once a map passes the established tests, it is considered to be acceptable.

In this paper, we analyze the problem of satisfactorily matching maps of adjacent areas and present a method for testing statistically whether a satisfactory match exists or not. The method has the advantage that it does not require field data to be used. The method is demonstrated using real data that refer to maps and diagrams compiled within the scope of the development of the Hellenic Cadastre (Lolonis 1997, Potsiou *et al.* 2001, Zentelis and Dimopoulou 2001).

The paper consists of six sections. In the next section, we define the problem formally. In the third section, we present the method and we develop the statistic that will be used to test satisfactory matching. In the fourth section, we illustrate the method using real world data from two cadastral survey studies that are under way in the island of Chios, Greece. In the fifth, we describe the techniques that must be used to ensure that the underlying prerequisites of the method are satisfied. In the sixth, we discuss briefly some special aspects of the method. Finally, in the last section, we summarize the conclusions.

2. PROBLEM DEFINITION

Assume that we have two maps that depict two adjacent regions: Region 1 and Region 2 (Figure 1). Also, assume that those regions are separated by a border line consisting of n well-identified points $\{1, \ldots, i, \ldots, n\}$. Let the set:

$$C = \{(X_1, Y_1), (X_2, Y_2), \ldots, (X_i, Y_i), \ldots, (X_n, Y_n)\} \tag{1}$$

be the set of true, but unknown, coordinates of those border points. The corresponding coordinates of those points on the map of Region 1 would be:

$$C' = \{(X_1', Y_1'), (X_2', Y_2'), \ldots, (X_i', Y_i'), \ldots, (X_n', Y_n')\} \tag{2}$$

and on the map of Region 2:

$$C'' = \{(X_1'', Y_1''), (X_2'', Y_2''), \ldots, (X_i'', Y_i''), \ldots, (X_n'', Y_n'')\}. \tag{3}$$

If there were no errors in the process of making the maps then

$$X_i' = X_i'' = X_i \tag{4}$$

and

$$Y_i' = Y_i'' = Y_i \tag{5}$$

for all $i=1,\ldots,n$. In general, however, these conditions would not hold and the corresponding coordinates would differ from each other. The question then is whether the observed differences are acceptable or not, based on the errors that are associated to the coordinates.

To answer this question, we would assume, first of all, that the coordinates of the points measured on the two maps are independent and normally distributed with mean the corresponding true coordinate value and variance σ^2. In mathematical notation, this statement is expressed as:

$$X_i' \sim N(X_i, \sigma^2) \wedge Y_i' \sim N(Y_i, \sigma^2) \wedge X_i'' \sim N(X_i, \sigma^2) \wedge Y_i'' \sim N(Y_i, \sigma^2) \tag{6}$$

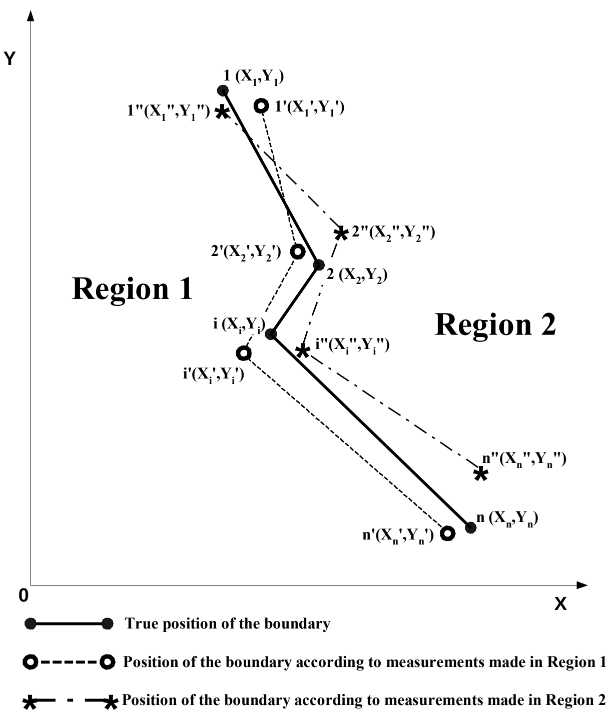
for $i=1,\ldots,n$.

The above assumption, although it is restrictive, is frequently adopted in applications where point coordinates result from field surveys or photogrammetric measurements. In this paper, it is adopted as a first approximation to handle the problem.

3. APPROACH TO SOLVE THE PROBLEM

In order to answer the above question, first, we must develop a statistic that would measure the overall mismatch of the two maps. Then, we must determine the distribution of that statistic. Finally, we can apply the statistic in each particular data set and obtain the answer.

Figure 1. Schematic representation of the position of a boundary of two adjacent regions according to measurements made on the map of each region



3.1 Development of the statistic that measures the mismatch between adjacent areas

If there were no errors in the coordinates of points depicted on the two maps and the maps were matching each other exactly, then, for a given point *i* along the boundary zone, we would have:

$$X_i^{'} = X_i^{"} = X_i \wedge Y_i^{'} = Y_i^{"} = Y_i \Leftrightarrow X_i^{'} - X_i^{"} = X_i - X_i^{"} = 0 \wedge Y_i^{'} - Y_i^{"} = Y_i - Y_i^{"} = 0 \Rightarrow$$

$$\left(X_i^{'} - X_i^{"}\right)^2 = 0 \wedge \left(Y_i^{'} - Y_i^{"}\right)^2 = 0 \Leftrightarrow \left(X_i^{'} - X_i^{"}\right)^2 + \left(Y_i^{'} - Y_i^{"}\right)^2 = 0 .$$

By letting $d_i^2 = \left(X_i^{'} - X_i^{"}\right)^2 + \left(Y_i^{'} - Y_i^{"}\right)^2$ we get $d_i^2 = \left(X_i^{'} - X_i^{"}\right)^2 + \left(Y_i^{'} - Y_i^{"}\right)^2 = 0$ (7)

This equation expresses the fact that when the maps of two adjacent areas match each other exactly, the Euclidean distance (squared) between each point *i'* that defines a particular feature i on the first map and the point i'' that defines the same feature on the other must be zero.

The converse of the above assertion is also true, that is, when condition (7) is true then the coordinates of a feature measured on the two maps would coincide. Indeed,

$$d_i^2 = \left(X_i^{'} - X_i^{''}\right)^2 + \left(Y_i^{'} - Y_i^{''}\right)^2 = 0 \Leftrightarrow \left(X_i^{'} - X_i^{''}\right)^2 = 0 \wedge \left(Y_i^{'} - Y_i^{''}\right)^2 = 0 \Leftrightarrow X_i^{'} = X_i^{''} \wedge Y_i^{'} = Y_i^{''} \quad (8)$$

Since equations (7) and (8) apply to all points $i=1...n$, we conclude that

$$Sd^2 = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} \left[\left(X_i^{'} - X_i^{''}\right)^2 + \left(Y_i^{'} - Y_i^{''}\right)^2\right] = 0 \quad (9)$$

By dividing all sides of equation (9) by the positive quantity $2\sigma^2$, where σ^2 is the variance of the coordinates (Equation 6), we get:

$$D^2 = \frac{Sd^2}{2\sigma^2} = \frac{1}{2\sigma^2}\left[\sum_{i=1}^{n} d_i^2\right] = \sum_{i=1}^{n}\left[\frac{\left(X_i^{'} - X_i^{''}\right)^2}{2\sigma^2} + \frac{\left(Y_i^{'} - Y_i^{''}\right)^2}{2\sigma^2}\right] = 0 \Leftrightarrow$$

$$D^2 = \sum_{i=1}^{n}\left[\left(\frac{X_i^{'} - X_i^{''}}{\sqrt{2}\sigma}\right)^2 + \left(\frac{Y_i^{'} - Y_i^{''}}{\sqrt{2}\sigma}\right)^2\right] = 0 \quad (10)$$

The variable D^2 is a measure of the degree of mismatch that exists between the two maps. Indeed, if the maps match each other perfectly, then the above variable would be zero. If they do not, the variable would take a positive value. In fact, the larger the mismatch is, the larger the value of D^2. Therefore, we would use D^2 as a suitable statistic for measuring and testing the mismatch between two adjacent areas depicted on two different maps.

3.2 Statistical distribution of D^2

Since we have assumed that $X_i^{'} \sim N(X_i, \sigma^2) \wedge X_i^{''} \sim N(X_i, \sigma^2) \Rightarrow$

$$\Delta x_i = X_i^{'} - X_i^{''} \sim N(0, 2\sigma^2). \quad (11)$$

This is true because Δx_i is a linear combination of two independent and normally distributed variables (e.g. Gnedenko, 1976, p. 146). The expected value of Δx_i is:

$$E[\Delta x_i] = E\left[X_i^{'} - X_i^{''}\right] = E\left[X_i^{'}\right] - E\left[X_i^{''}\right] = X_i - X_i = 0$$

The variance of Δx_i is:

$$\begin{aligned}
\operatorname{var}\Delta x_i &= E[\Delta x_i - E[\Delta x_i]]^2 = E\left[(\Delta x_i)^2 - 2(\Delta x_i)E[\Delta x_i] + (E[\Delta x_i])^2\right] \\
&= E\left[\left(X_i^{'} - X_i^{''}\right)^2 - 2\left(X_i^{'} - X_i^{''}\right)0 + 0^2\right] = E\left[\left(X_i^{'} - X_i^{''}\right)^2\right] \\
&= E\left(X_i^{'2}\right) - 2E\left[X_i^{'}X_i^{''}\right] + E\left[X_i^{''2}\right] \\
&= E\left[X_i^{'2}\right] + E\left[X_i^{''2}\right] = \operatorname{var}X_i^{'} + \operatorname{var}X_i^{''} \qquad \text{(Due to the zero covariance between } X_i^{'} \text{ and } X_i^{''}\text{)}
\end{aligned}$$

$= \sigma^2 + \sigma^2 \Rightarrow$ (Assumption of equal variances of $X_i^{'}$ and $X_i^{''}$)

$\text{var}\Delta x_i = 2\sigma^2$.

From equation (11) we can infer then that the variable:

$$Z_{x_i} = \frac{X_i^{'} - X_i^{''}}{\sqrt{2}\sigma}$$

would be a standard normal variable and, consequently, its square would be a *Chi-square* variable with one degree of freedom (e.g. Hoel, 1984, p. 136). In mathematical notation, this fact is represented as:

$$d_{X_i}^2 = \left(\frac{X_i^{'} - X_i^{''}}{\sqrt{2}\sigma} \right)^2 \sim \chi^2(1) \tag{12}$$

Similarly, $\Delta y_i = Y_i^{'} - Y_i^{''} \sim N(0, 2\sigma^2)$ and, therefore, the variable:

$$d_{Y_i}^2 = \left(\frac{Y_i^{'} - Y_i^{''}}{\sqrt{2}\sigma} \right)^2 \sim \chi^2(1) \tag{13}$$

Since $d_{X_i}^2$ and $d_{Y_i}^2$ are independent χ^2 variables, we can infer that their sum also would be a χ^2 variable. The degrees of freedom of this new variable would be the sum of the degrees of freedom of the summand variables (e.g. Hoel, 1984, p. 135). Thus, the variable:

$$\left(\frac{d_i}{\sqrt{2}\sigma} \right)^2 = \left(\frac{X_i^{'} - X_i^{''}}{\sqrt{2}\sigma} \right)^2 + \left(\frac{Y_i^{'} - Y_i^{''}}{\sqrt{2}\sigma} \right)^2 \sim \chi^2(2) \tag{14}$$

Similarly, if we extend the above notion for all *n* points in the sample:

$$D^2 = \sum_{\iota=1}^{n} \left(\frac{d_i}{\sqrt{2}\sigma} \right)^2 = \sum_{\iota=1}^{n} \left[\left(\frac{X_i^{'} - X_i^{''}}{\sqrt{2}\sigma} \right)^2 + \left(\frac{Y_i^{'} - Y_i^{''}}{\sqrt{2}\sigma} \right)^2 \right] \sim \chi^2(2n) \tag{15}$$

Thus, the statistic D^2 that measures the overall mismatch between two maps follows the χ^2 distribution with *2n* degrees of freedom.

3.3 Use of D^2 in statistical inference

A necessary and sufficient condition for the two maps to match exactly each other is:

$D^2=0$.

Thus, in order to test satisfactory matching of such maps, in cases where there is random error in the coordinates of points, it suffices to test the hypothesis:

$H_0: D^2=0$ against the alternative $H_1: D^2>0$.

If, for a particular set of random data and confidence level p, we get $D^2 \leq \chi_p^2(2n)$ then we accept the hypothesis H_0 that the two maps match each other satisfactorily. Otherwise, we reject H_0 and conclude that the matching is not satisfactory.

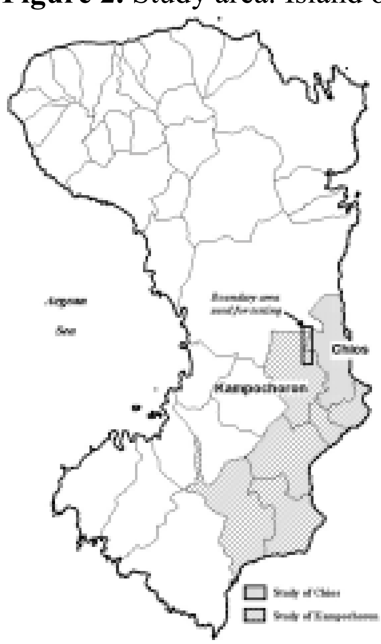
It must be noted that, although the method is described presuming that the points used in computing the statistic lie on the boundary of the two adjacent regions, this restriction is not mandatory. Careful examination of the equation reveals that the only restriction that exists is that the points must be displayed on both maps, as it is the case when the two maps overlap.

4. ILLUSTRATION OF THE METHOD

4.1 Data from the cadastral survey studies in the island of Chios, Greece

The method presented in this paper will be illustrated using data from two cadastral survey studies that are under way in the island of Chios, Greece. Specifically, the first study aims at the development of a cadastral database in the municipality of Chios, while the second in the adjacent municipality of Campochoron (Figure 2). The contractors who have undertaken the task to develop the initial cadastral database have compiled 1:5.000 scale orthophotomaps of the area (Figure 3) in order to delineate land parcels on them and then associate the geometric information with the ownership data through the Parcel Code Identification Number. Since those cadastral survey projects run more or less in parallel, the orthophotomaps that are compiled by one contractor do not necessarily match exactly with orthophotomaps compiled by another. The question that the Quality Control Division of Ktimatologio S.A. (Hellenic Cadastre) has to answer is whether the two sets of maps that are submitted by the contractors meet the accuracy specifications in terms of satisfactory matching.

Figure 2. Study area. Island of Chios, Greece



Using the method described in Section 3, we have selected randomly a set of 25 well-identified features that appear on the overlapping areas and along the boundary of the two adjacent municipalities. Then, we have measured the coordinates X, Y of those points on each map (Table 1, Columns 3-6). Those coordinates are in meters and are expressed into the Hellenic Geodetic Reference System '87 (HEMCO, 1987). Then, by letting **σ=0,78m,** we have computed the individual terms: $(X'-X'')/\sqrt{2}\sigma$ and $(Y'-Y'')/\sqrt{2}\sigma$ of Equation (10) for each observation point (Table 1, Columns 7-8). Consequently, we have added the squares of the above terms to create the summands of Equation (10) (Table 1, Column 9). The sum of those values gives us the value of D^2 for this particular sample, that is: $D^2=32,7167$.

Figure 3. Orthophotomaps of the study area and indicative observed points

Map 1. Cadastral study of Campochoron. Features measured on this map are indicated by the center of the red circles

Map 2. Cadastral study of Chios. Features measured on this map are indicated by the center of the yellow circles

It must be noted that the value for the standard error of measurements that we have used here (σ=0,78m) results from the Technical Specifications of the Hellenic Cadastre, which specify that the tolerance value for the 1:5.000 scale maps at the 99% confidence level is 2 meters (Ktimatologio, 1997, Ch. 4, Article 8, par. 8.2).

Given the fact that the number of points in the sample is n=25, we conclude from equation (15) that the statistic $D^2=32,7167$ that we have computed corresponds to the χ^2 value with $n'=2*n$=50 degrees of freedom. Since the degrees of freedom exceed 30 and, therefore, the corresponding value of the χ^2 distribution is not listed on the standard statistical tables (e.g Hoel 1984), we compute instead the normal deviate:

$$z_D = \sqrt{2D^2} - \sqrt{2n' - 1} \quad (16)$$

to determine whether the value of D^2 falls into the critical region at a given confidence level, let's say 95%.

Table 1. Observations made on the orthophotomaps of the two studies in Chios, Greece

S/N	Description	X' (m)	Y' (m)	X" (m)	Y" (m)	Ndx= (X'-X")/ $\sqrt{2}\sigma$	Ndy= (Y'-Y")/ $\sqrt{2}\sigma$	D_i^2= Ndx^2+Ndy^2
1	Center of bush	683.807,42	4.249.029,40	683.806,10	4.249.027,58	1,1966	1,6499	4,1542
2	Center of tree	683.740,48	4.248.856,50	683.739,48	4.248.853,53	0,9065	2,6924	8,0711
3	Corner of road intersection	683.800,80	4.248.710,43	683.800,13	4.248.709,77	0,6074	0,5983	0,7269
4	Center of bush	683.853,16	4.248.641,53	683.852,83	4.248.641,53	0,2992	0,0000	0,0895
5	Center of rock	683.893,43	4.248.566,84	683.893,43	4.248.566,51	0,0000	0,2992	0,0895
6	Corner of intersecting roads	683.943,93	4.249.246,77	683.943,63	4.249.246,02	0,2720	0,6799	0,5362
7	Center of bush	683.536,42	4.248.804,73	683.535,76	4.248.803,08	0,5983	1,4958	2,5954
8	Center of rock	683.944,91	4.248.519,03	683.945,26	4.248.518,36	-0,3173	0,6074	0,4696
9	Center of rock	683.881,66	4.248.307,27	683.881,55	4.248.307,13	0,0997	0,1269	0,0261
10	Point of stone wall	683.864,00	4.248.307,24	683.863,74	4.248.306,89	0,2357	0,3173	0,1562
11	Center of bush	683.543,07	4.248.539,67	683.543,86	4.248.541,70	-0,7162	-1,8403	3,8996
12	Center of bush	683.548,42	4.248.541,66	683.547,88	4.248.539,67	0,4895	1,8040	3,4942
13	Corner of building	683.662,10	4.248.233,76	683.662,12	4.248.231,52	-0,0181	2,0307	4,1239
14	Center of rock	683.677,51	4.248.286,05	683.676,93	4.248.284,91	0,5258	1,0335	1,3445
15	Center of rock	683.869,38	4.248.130,42	683.869,47	4.248.130,42	-0,0816	0,0000	0,0067
16	Center of fold	683.912,82	4.248.032,61	683.912,80	4.248.032,53	0,0181	0,0725	0,0056
17	Center of bush	683.789,15	4.247.774,97	683.788,04	4.247.775,30	1,0063	-0,2992	1,1021
18	Center of bush	683.798,36	4.247.774,31	683.798,17	4.247.774,88	0,1722	-0,5167	0,2967
19	Center of rock	683.938,66	4.247.489,15	683.938,32	4.247.489,65	0,3082	-0,4533	0,3005
20	Center of bush	683.636,48	4.247.393,10	683.636,15	4.247.393,11	0,2992	-0,0091	0,0896
21	Center of bush	683.544,50	4.247.284,58	683.544,67	4.247.284,42	-0,1541	0,1450	0,0448
22	Center of bush	683.603,22	4.247.086,57	683.602,78	4.247.086,78	0,3989	-0,1904	0,1953
23	White point in a bush	683.693,64	4.246.958,63	683.693,43	4.246.958,68	0,1904	-0,0453	0,0383
24	Junction of stonewalls	683.889,13	4.246.116,98	683.888,90	4.246.116,62	0,2085	0,3264	0,1500
25	Center of bush	683.577,14	4.249.054,06	683.577,93	4.249.054,55	-0,7162	-0,4442	0,7102
	D^2							**32,7167**

Where σ=0,78m. This value corresponds to the standard error of the 1:5.000 scale orthophotomaps compiled in cadastral surveys in Greece.

Indeed, substituting the corresponding values into equation (16) we get:

$$z_D = \sqrt{2D^2} - \sqrt{2(2n)-1} = \sqrt{2*32,7167} - \sqrt{2*(2*25)-1} = -1,86 < 1,645 = z_{p<0.95}$$

Thus, we conclude that D^2 does not fall into the critical region and, therefore, we accept, for this case, the hypothesis H_o stating that the two maps match each other satisfactorily. If the value of z_D were greater than 1,645, then we would reject the hypothesis H_0 and we would conclude that the two maps don't match satisfactorily.

5. TEST FOR THE FULFILLMENT OF THE UNDERLYING ASUMPTIONS

The method described in Section 3 is based on the assumption that the observed errors are random and there is no systematic pattern in them. In many instances, however, it is likely that such systematic patterns may exist in the deviations of *X'*, *X"*, *Y'*, and *Y"*. This can happen, for instance, when the two maps in question are translated, rotated, or scaled relative to each other. In this section, we would demonstrate how this problem can be detected and how it can be overcome. For simplicity reasons, we would examine the case in which the systematic patterns in the deviations of the *Xs* and *Ys* are caused by translations and rotations only and not by differences in scale[1].

The equations that relate two coordinate sets that are subject to translation and rotation transformations are:

$$X^{''} = X_0 + X^{'}\cos\theta + Y^{'}\sin\theta \qquad (17)$$

$$Y^{''} = Y_0 + X^{'}(-\sin\theta) + Y^{'}\cos\theta \qquad (18)$$

where X_0 and Y_0 represent the relative shift of the two coordinate sets and θ represents the corresponding rotation angle. The variables X', Y', X'', Y'' are known because they are measured on the maps. The parameters X_0, Y_0, and θ are unknown and must be determined. The determination of those values could be made using multivariate regression analysis.

If there is no shift or rotation in the dataset, the parameters would take the values: $X_0=0$, $Y_0=0$ and $\theta=0$. However, if there is such an effect, then at least one of the above parameters would be different from zero and the overall regression would be significant at a pre-specified critical value (e.g. Johnston 1984, p. 186). These notions give us then the means to examine and test whether there are statistically significant translations or rotations in a dataset.

By letting, for simplicity reasons, $X_0=a$, $Y_0=b$, $\cos\theta=1-c$, and $\sin\theta=d$ and substituting those values into the equations (17) and (18) we get:

$$X^{''} = 1*a + 0*b + X^{'}*(1-c) + Y^{'}*d \Rightarrow X^{''} - X^{'} = 1*a + 0*b + (-X^{'})*c + Y^{'}*d \qquad (19)$$

$$Y^{''} = 0*a + 1*b + Y^{'}*(1-c) + (-X^{'})*d \Rightarrow Y^{''} - Y^{'} = 0*a + 1*b + (-Y^{'})*c + (-X^{'})*d \qquad (20)$$

Equations (19) and (20) are the *observation equations* of the regression analysis and can be used to estimate the parameters *a*, *b*, *c*, and *d* of the model[2]. Using the data of Table 1 as input, we obtain the results shown in Table 2.

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[1] When, in addition to translations and rotations, there are scale differences between the two maps, the problem can be treated by expanding the corresponding equations into Taylor series and applying Least Squares procedures to the resulting equations.

[2] It must be noted that we could have specified a more efficient regression model having only three parameters. Something like that, however, would require linearization of the equations using Taylor series and iterative procedures to perform regression analyses. This detailed, yet more formal, technique goes beyond the illustrative purposes of this section of the paper.

Table 2. Results of the regression analysis

R Square	664	Adjusted R^2	0,026469599	Std Error	0,961412315	Obs :50
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	4	5,066972592	1,266743148	1,370468955	0,25924335	
Residual	46	42,51842742	0,924313639			
Total	50	47,58540001				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
a=X0	-756,8442719	1113,201485	-0,67988076	0,499987369	-2997,600499	1483,911955
b=Y0	-2221,167186	1113,201483	-1,995296647	0,051953206	-4461,923409	19,58903639
c=1-cosθ	-0,00053774	0,000258715	-2,078502483	0,043268688	-0,001058506	-1,6974E-05
d=sinθ	9,16716E-05	0,000258715	0,354334366	0,724707524	-0,000429094	0,000612437

The above results show that the *F-statistic* of the overall regression is 1,37 and is insignificant at the 95% confidence level. Thus, we can conclude that there is no significant presence of translation or rotation in the dataset of Table 1.

6. DISCUSSION

The method presented in Section 3 must be used in conjunction with the regression analysis described in Section 5. This practice would safeguard against systematic spatial trends in the data sets. It is conceivable that there might be slight translations and rotations in the data which may be detected by the regression analysis but not by the *Chi-square* test. Conversely, the spatial pattern of the discrepancies might be scattered in such a way that the regression is insignificant and the *Chi-square* statistic significant. Of course, both techniques cannot guarantee that there is no existence of simultaneous translation, rotation, or scaling of both areas with respect to the absolute reference system. In order to test for such a possibility there is a need for exogenous data.

7. CONCLUSIONS

In this paper, we have described a method that can be used to test whether two maps that have been compiled independently and depict adjacent areas match each other satisfactorily. This method is based on a statistic which measures the overall mismatch between the two maps and which is a function of direct measurements made on those maps and the standard error of those measurements. It is shown that the statistic follows the χ^2 distribution with $2*n$ degrees of freedom, where n is the number of points measured on the maps. The method was illustrated using data from the 1:5.000 scale orthophotomaps compiled within the scope of the development of the Hellenic Cadastre in two indicative adjacent municipalities of the island of Chios in Greece. This method could be applied in many other similar settings that aim at testing satisfactory edge matching of independently made maps of adjacent areas.

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DISCLAIMER

The views presented in this paper are personal and do not necessarily represent the official views of *Ktimatologio S.A*.

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