Introduction to Vertical Reference Frames

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Outline

- Why do we need a vertical reference system?
  - Basic components
  - Types of reference surface
  - Types of heights
  - Types of tidal systems
- Geometrical and physical heights
- Global VRS - IHRS
- Regional European VRS
- Local realization in Poland

Why do we need a reference system?

- In geodesy we are describing Earth's: **shape**, **gravity field** and **orientation in space** with **changes** of this components **in time**.
- For many tasks we use GNSS positioning while requiring physical heights
- We need to know the relation between heights from different techniques
- **what elements we need to define to prescribe a "Vertical reference system"?**
  - Basic components
  - Reference surface
  - Height system

  } related
  - Unit(s)
  - Tidal system

Basic components

- Geopotential numbers – C_P $[^{m^2}/_{s^2}]$
  - difference between potential of the Earth's gravity field at reference surface and potential at measured point:

$$\boldsymbol{C_P = -\Delta W_P = W_0 - W_P}$$

- Heights (physical) – H $[m]$
  - geopotential number divided by value of gravity along the plumb line, depends on the kind of physical height system

$$\boldsymbol{H_P = \frac{C_P}{g'}}$$

$g' - value\ of\ Earth's\ gravity$

- Heights (geometric) – h $[m]$
  - distance form reference ellipsoid to point, measured along the normal to ellipsoid, depends on mathematical definition of ellipsoid

Types of reference surfaces

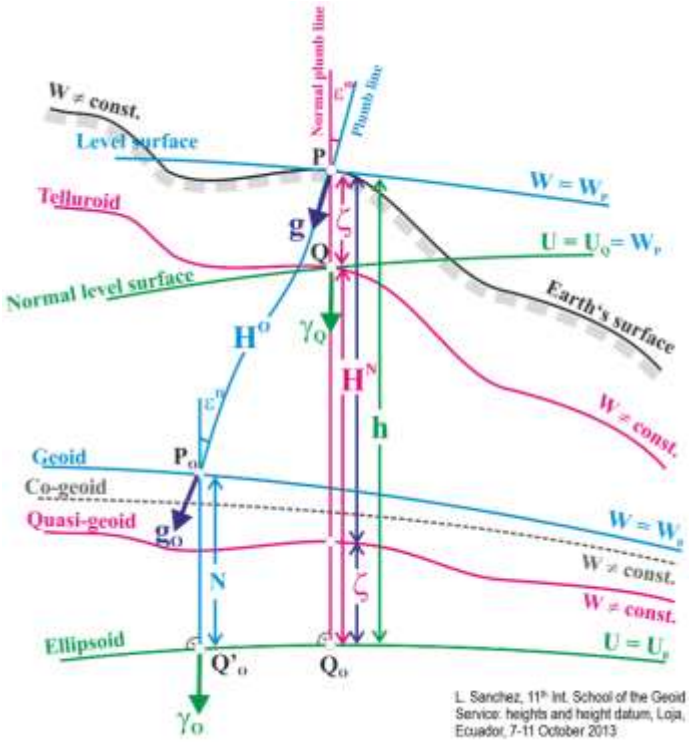
- **ellipsoid** – mathematical model of the Earth which closely approximates the geoid

- **geoid** – equipotential surface of Earth's gravity field which best fits mean sea level at a certain epoch. Mean sea level can be referred to one (usually local) or more (global) tide gauge.

- **quasi-geoid** – not equipotential surface, closely related to normal heights, normal gravity at reference ellipsoid and Molodenskii's Boundary Value problem.
  - "The height anomaly ζ is the distance, along the normal plumb line, between the Earth's surface and the telluroid. When plotted above the ellipsoid the resulting surface is called the quasi-geoid"

[C. Rizos, Technical Seminar Vertical references frame in Practice, Singapore 27-28 July 2015]

Types of heights



Height system

PHYSICAL

ORTHOMETRIC (H^O) – distance from **geoid** to point, measured along the plumb line (with known real **gravity g** inside the masses, along the plumb line)

HELMERT ORTHOMETRIC – orthometric heights where mean gravity is calculated using a Prey reduction with standard density $\rho_0 = 2.67$

NORMAL ORTHOMETRIC – orthometric heights computed with same density around the point and with using mean normal gravity (this is NOT a normal heights)

NORMAL (H^N) – distance from **quasi-geoid** to point, measured along the normal plumb line (with known **normal gravity γ,** associated with **reference ellipsoid**)

DYNAMIC – distance from geoid to point, measured along the plumb line (with **constant normal gravity** γ^{45}at 45°,associated with **reference ellipsoid**, have no geometrical meaning)

GEOMETRICAL

GEOMETRICAL (**h**)– distance form reference ellipsoid to point, measured along the normal to ellipsoid

Types of tidal systems

Tide system is defined to include the permanent surface deformation caused by the Sun and the Moon. There are 3 concepts to deal with it:

- **None-tidal** – the permanent deformation is eliminated from the shape of the Earth; From the potential field quantities (gravity, geoid etc) both **the tide-generating potential**, and **the deformation potential** of the Earth (the indirect effect) **are eliminated.**
- **Mean tidal** – the permanent effect is not removed from the shape of the Earth; the shape therefore corresponds to the long-time average under tidal forcing; **The potential field retains the potential of this average Earth**, and also the **time-average of the tide-generating potential** (though it is not due to the masses of the Earth)
- **Zero tidal** – **eliminates the tide-generating potential but retains its indirect effect**, i.e., the potential of the permanent deformation of the Earth. The gravity field is generated only by the masses of the Earth (plus the centrifugal force).

[J.Mäkinen, Symposium of the IAG Subcommission for Europe (EUREF) Brussels, 18-21.06.2008]

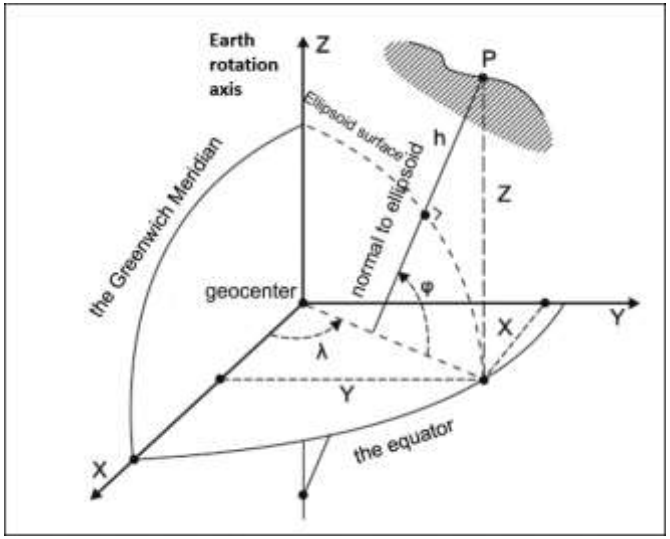
Geometrical and physical heights

Geometrical Heights in GRS80

- Basic component – h [m]
- Reference surface – reference ellipsoid GRS80
  - 4 defining parameters:
    - Semi-major axis: $a = 6\,378\,137\ m$
    - Geocentric gravitational constant: $GM = 3986005 \times 10^8\ {}^{m^3}/_{s^2}$
    - Dynamical form factor: $J_2 = 108\,263\ \times 10^{-8}$
    - Angular velocity of rotation: $\omega = 7\,292\,115 \times 10^{-11} s^{-1}$
- Normal gravity γ:

$$\gamma = \frac{a\gamma_a \cos^2 \varphi + b\gamma_b \sin^2 \varphi}{\sqrt{a \cos^2 \varphi + b \sin^2 \varphi}}$$ *(Somigliana 1929)*

  - semi-minor axis: $b = 6\,356\,752.3141\ m$
  - $\gamma_a = 9.7803267715 \frac{m}{s^2}$ - normal gravity at Equator
  - $\gamma_b = 9.8321863685 \frac{m}{s^2}$ - normal gravity at Pole
- Tidal system – non-tidal



Geometrical and physical heights

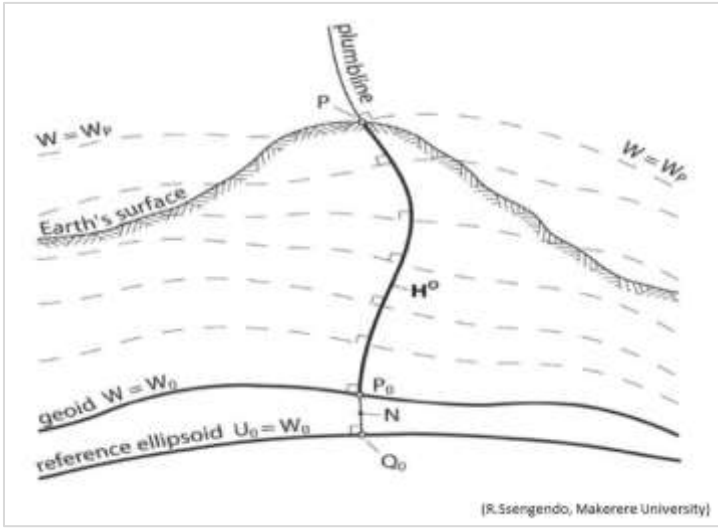
Orthometric heights

- Basic component - $H_P^O = \frac{C_P}{\bar{g}_P}$,

  $\bar{g}$ - the average value of gravity along the plumb line

- To compute H_P^O we should know the average value of gravity along the plumb line, which depends on density inside the masses and topography around point.
  - In nowadays, the models created from surface gravity measurements can be used as well.
- Reference surface - geoid
- To calculate the H_P^O from geometrical height we should know N - the distance between the geoid and the ellipsoid:

$$N = h - H^O$$



Geometrical and physical heights

Normal heights - the geometric distance between the ellipsoid and telluroid along the normal plumb line:

- Basic component - $H_P^N = \frac{C_P}{\bar{\gamma}_P}$,

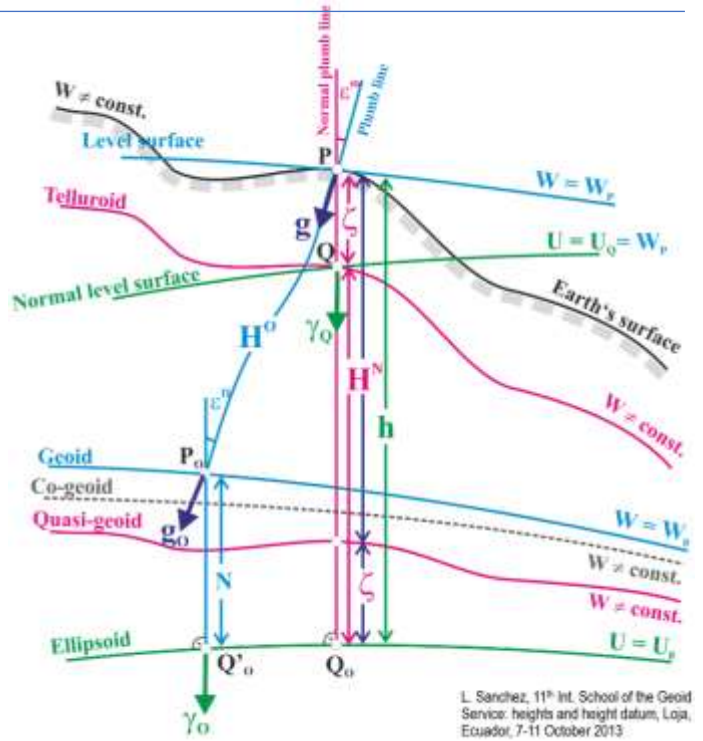
$\bar{\gamma}$ - the average value of normal gravity

$$\bar{\gamma}_P = \gamma_0 \left(1 - \frac{1}{a}(1 + f + m - 2f \sin^2 \varphi) H^N + \cdots\right)$$

- The telluroid is defined as the surface at which the normal potential U_Q is equal to real potential W_P at the Earth's surface
- Reference Surface – quasi-geoid
- In normal heights we use height anomaly ζ instead of geoid height N:

$$\zeta = h - H^N$$

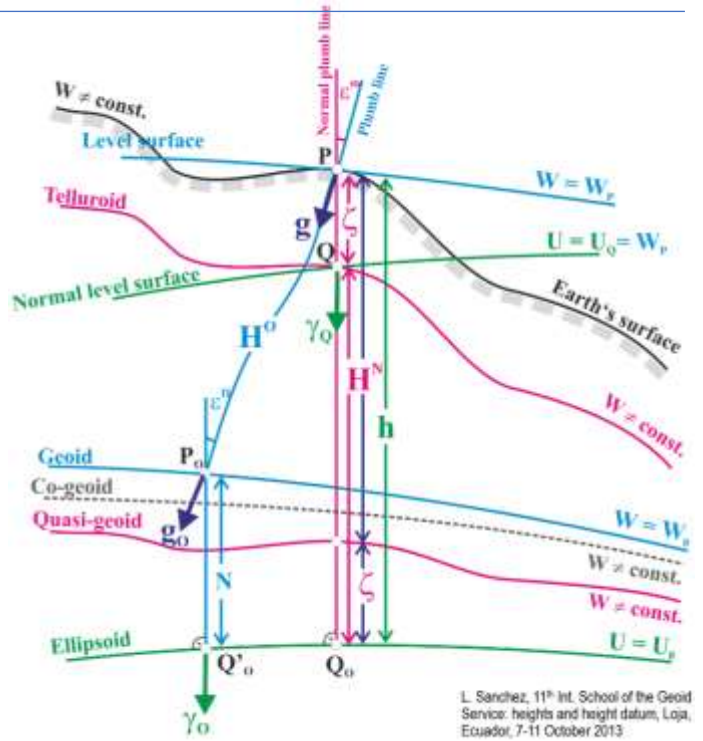
- This heights and height anomalies could be calculated without knowledge of the topographic density



Geometrical and physical heights

Relation between heights from different techniques

- To compute physical heights from geometrical heights we must know:
  - Kind of height and reference surface in physical heights
  - Reference ellipsoid in geometrical heights
  - Geodetic reference frame
- All models of geoids and quasi-geoids are strictly associated with particular terrestrial and vertical frame!
- Earth is in still movement – we should know the epoch of the measurements and velocities of control points!



Global VRS - IHRS

In **2015** General Assembly of the International Union of Geodesy and Geophysics (IUGG), the International Association of Geodesy (IAG) released a **resolution for the definition and realization** of an **International Height Reference System (IHRS):**

- the vertical coordinates are geopotential numbers:

$$-\Delta W_P = \boldsymbol{C_P} = W_0 - W_P$$

referring to the equipotential surface of the Earth's gravity field realized by the conventional value

$$\boldsymbol{W_0 = 62\,636\,853.4\ \frac{m^2}{s^2}}$$

- the spatial reference of the position P for the potential

$$W_P = W(\boldsymbol{X}_{P\ in\ ITRF})$$

is given by coordinates X of the **International Terrestrial Reference Frame (ITRF).**

- parameters, observations and data related to the **mean tidal system/mean crust**
- The estimation of **X_P, W_P (or C_P)** includes their **variation with time**;
- The unit of length is the **meter** and the unit of time is the **second** (SI).

[L.Sánchez, Workshop for the Implementation of the GGRF in Latin America, Buenos Aires,16-20.09.2019]

Global VRS - IHRS

TWO WAYS IN REALIZATION OF REFERENCE FRAME

PHYSICALLY
SOLID MATERIALIZATION OF POINTS/OBSERVING INSTRUMENTS

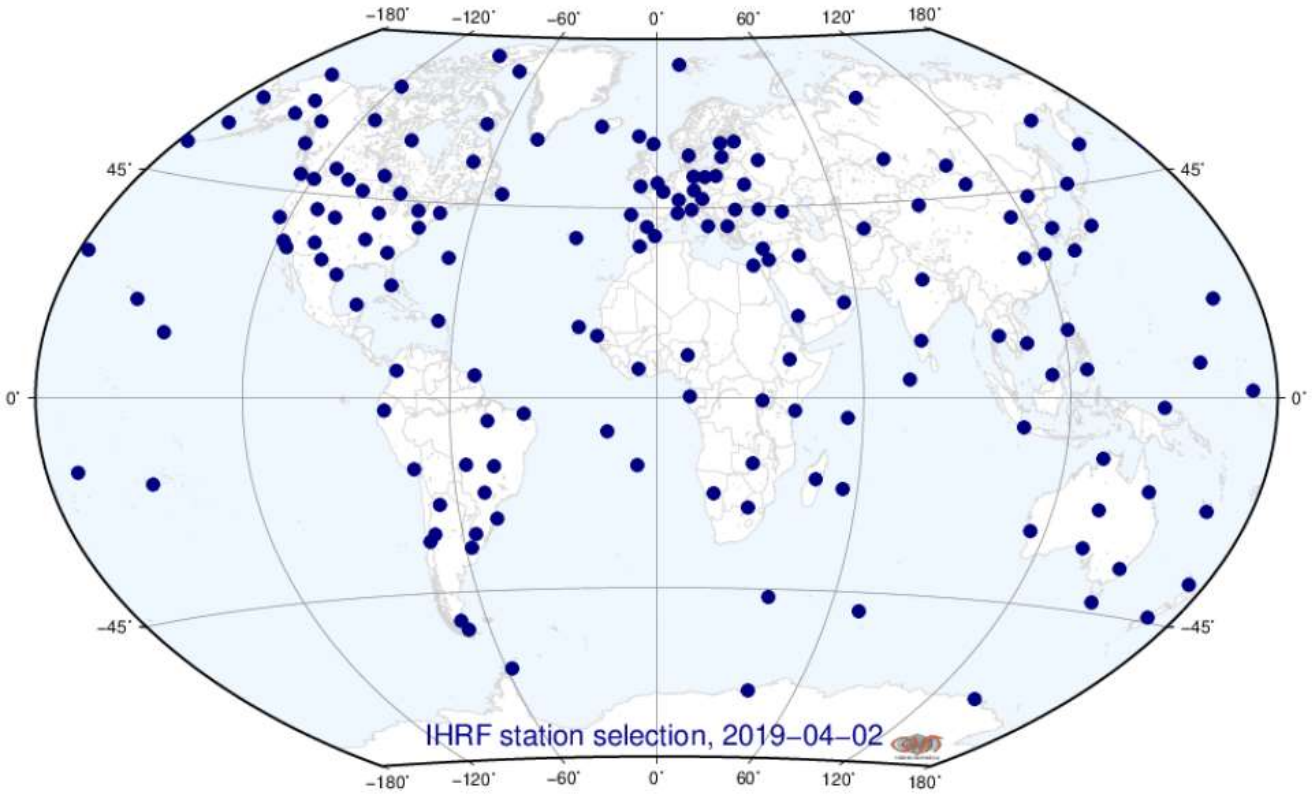
MATHEMATICALLY
THE DETERMINATION OF COORDINATES REFERRING TO REFERENCE SYSTEM

- The coordinates of the points are computed from the measurements following the definition of the reference system.

- The actions taken since 2015:
  - Establishment of a global reference network for the IHRS realization: *the International Height Reference Frame (IHRF)* (04.2019)
  - Evaluation of different strategies for the determination of potential values as IHRS/IHRF reference coordinates (main action the Colorado experiment)
  - Identification of required standards, conventions and procedures needed to ensure consistency between the definition (IHRS) and the realisation (IHRF)
  - Strategy for the integration (transformation) of existing vertical datums into the IHRS/IHRF (Sánchez and Sideris, 2017)

[L.Sánchez, Workshop for the Implementation of the GGRF in Latin America, Buenos Aires,16-20.09.2019]

Global VRS - IHRS



Preliminary reference network for the IHRF:
170 stations welldistributed worldwide, materialized by GNSS continuously operating stations and co-located with:

- VLBI (30 sites),
- SLR (40 sites),
- DORIS (35 sites),
- absolute gravity – IGRF (77 sites),
- tide gauges (26 sites),
- national levelling networks (23 sites).

[L.Sánchez, Workshop for the Implementation of the GGRF in Latin America, Buenos Aires,16-20.09.2019]

Global VRS - IHRS

Planned activities (2019-2023):

- Based on the Colorado experiment outcomes, to elaborate a document with detailed standards and conventions for the realization and maintenance of the IHRS.
- With the support of the IAG Commission 2, the IGFS and the ICCT to promote the study of
  - quality assessment in the determination of potential values;
  - determination of potential changes with time $\dot{W}$;
  - realization of the IHRS in marine areas.
- In agreement with the IGFS and the IAG Commission 2, to design a strategy to install an operational infrastructure within the IGFS to ensure the maintenance and availability of the IHRF in a long-term basis.
- More details about *Working Group 0.1.3: Implementation of the International Height Reference Frame (IHRF)* activity:

https://ggos.org/about/org/fa/unified-height-system/wg/ihrf-implementation/

[L. Sánchez, R. Barzaghi, Activities and plans of the GGOS Focus Area Unified Height SystemEGU General Assembly 2020, EGU2020-8625]

Regional VRS – pan-European VRS

- About EVRF and its realizations
  - Several projects for unifying vertical networks have already been developed in **50-70 years of XX century.** Due to the political division of Europe, the projects were implemented separately for the western and eastern parts of the continent.
  - In 90's (1994-1995) the project was resumed and in four years more countries joined the **United European Leveling Network**. First results of adjusted European heights were handed over to each participating country in **1999** (named **EVRF2000**).
  - Simultaneously to the UELN adjustment, **definitions and standards** of the European Vertical Reference System (EVRS) were developed and was adopted at the **EUREF symposium 2000 in Tromsø.**
  - In XXI century we have 2 more realizations of EVRS:
    - **EVRF2007** – computed after new data sets were provided by participaiting countries. In 2008 EUREF Symposium proposes to the European Commission that EVRF2007 shall be used as the vertical reference for pan-European geo-information and ithas been included in INSPIRE directive.
    - **EVRF2019** – another new data sets were available and in 2015 started a new realization of EVRS.
  - **Pan-European network is an integrated network of GNSS, leveling and tide gauge observations.**

Regional VRS – pan-European VRS

- EVRS definition
  - a kinematic height reference system
  - **$W_0 = W_{0E}$ = const.** and which is in the level of the **Normaal Amsterdams Peil**
  - The units:
    - length - the **meter** (SI).
    - time - **second** (SI).
    - This **scale** is consistent with the **TCG time coordinate** for a geocentric local frame
  - height components are the differences between the potential W_P of the Earth gravity field through the considered points P, and the potential W_{0E} of the EVRS conventional zero level.
    - The potential difference $-\Delta W_P$ is also designated as the geopotential number c_P:

$$-\Delta W_P = c_P = W_{0E} - W_P$$

  - The EVRS is a zero tidal system
  - to convert the geopotential numbers to normal heights, a normal gravity field and geodetic latitude is required. The GRS80 normal gravity field is adopted for the purpose, evaluated at ETRS89 coordinates.
  - Normal gravity at the ellipsoid is computed from the Gravity Formula 1980 (Moritz H., 1980) using the series expansion

Regional VRS – pan-European VRS

- EVRS definitions (cont.)
  - to convert the geopotential numbers to normal heights is adopted:
    - The **GRS80** normal gravity field
    - evaluated at **ETRS89** coordinates.
  - Normal gravity at the ellipsoid is computed from the Gravity Formula 1980 (Moritz H., 1980) using the series expansion

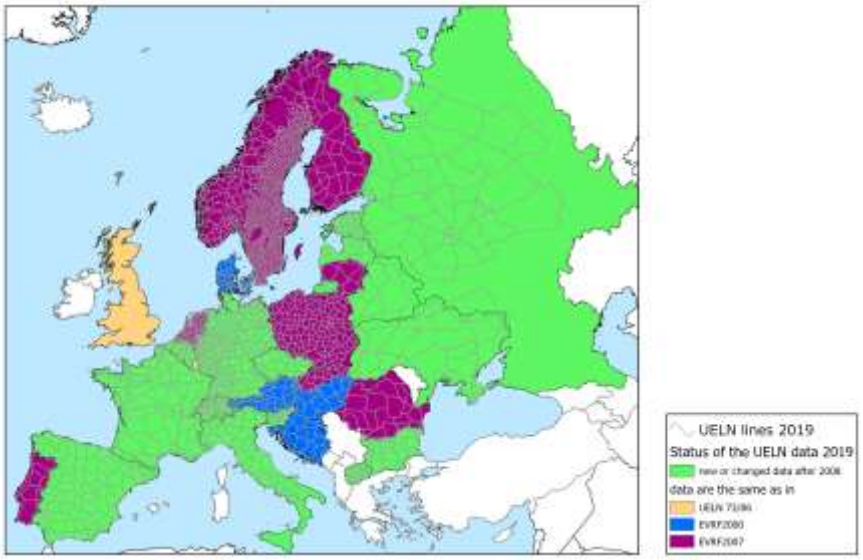
$$\gamma_0 = 9.783\,267\,715\left(1 + 0.005\,279\,0414\sin^2\varphi + 0.000\,023\,2718\sin^4\varphi + 0.000\,000\,1262\sin^6\varphi + 0.000\,000\,0007\sin^8\varphi\right)\ \mathrm{m\,s^{-2}} \qquad (1)$$

  - The average value of the normal gravity along the normal plumb line is determined by the formula:

$$\bar{\gamma} \approx \bar{\gamma}_H = \gamma_0\left[1 - \left(1 + f + m - 2f\sin^2\varphi\right)\frac{H}{a} + \frac{H^2}{a^2}\right] \qquad (2)$$

where H is an approximate value for H_P and γ is from equation (1). The notation and the numerical values for the other quantities are according to (Moritz H., 1988)

Regional VRS – pan-European VRS



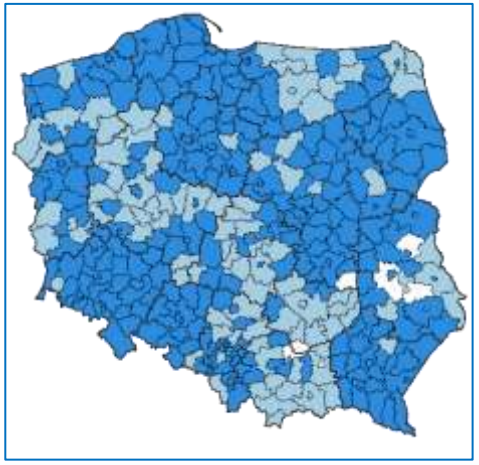
- More information about European Vertical Reference System could be found at:

  https://evrs.bkg.bund.de/Subsites/EVRS/EN/Home/home.html

Local realization in Poland

Vertical frames in Polish law:

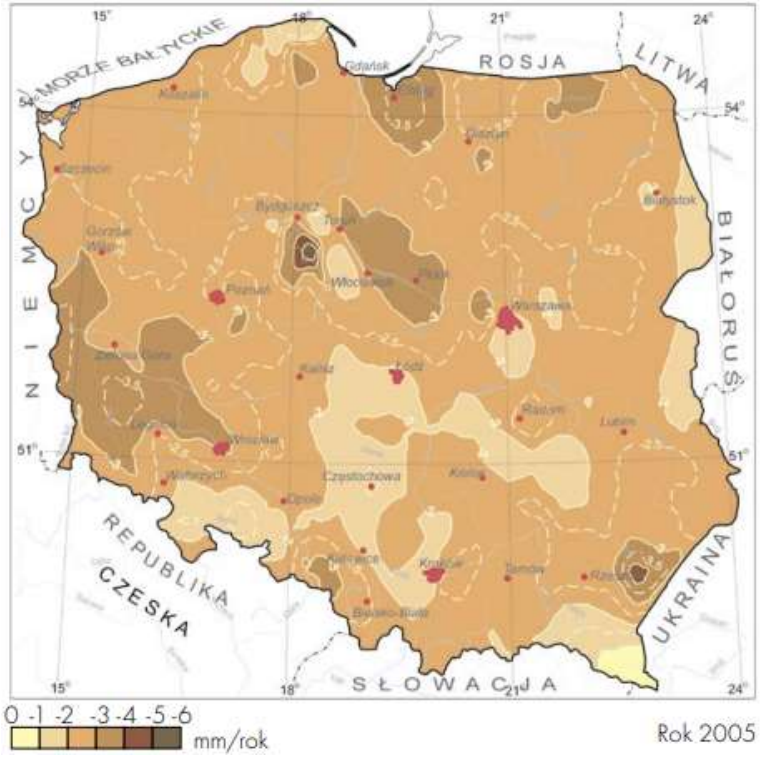
- According to Regulation on the national spatial reference system (2012 r.) in Poland 2 vertical reference frames could be in use:
  - **PL-KRON86-NH** – normal heights, level of the Baltic Sea Level in Kronstadt – can be used not longer than **31.12.2023 r.**
  - **PL-EVRF2007-NH** – normal heights, Amsterdam Nodal Point
- Polish leveling network of **1st and 2nd** class (fundamental and basic class) was published in PL-EVRF2007-NH at **2014**.
- From that year all counties are implementing the system at their area in leveling network of **3rd** class:



PL-EVRF2007-NH implementation status	September 2022	
	Number of Counties	%
Not implemented (white)	7	1,84 %
Implementation in progress (light blue)	99	26,05 %
Implemented in County (blue)	**274**	**72,11 %**

Local realization in Poland

- MODEL OF THE VERTICAL MOVEMENTS EARTH'S CRUST AN AREA OF POLAND
  - For the area of Poland such movements was calculated twice by Tadeusz Wyrzykowski from the Institute of Geodesy and Cartography in Warsaw – in 1961 (2nd levelling campaign) and 1986 (3rd levelling campaign).
  - After 4'th levelling campaign, in 2005, **PhD Kamil Kowalczyk** from The University of Warmia and Mazury in Olsztyn (UWM) was computed **new model** of the vertical movements earth's crust an area of Poland Mean velocity is from -1,5 to -3 mm/year.



[PhD K.Kowalczyk, Vertical movements of the earth's crust in Poland, 2006]

Thank You for attention...