Mathematical model and results of a new positioning algorithm

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ABSTRACT

A new mathematical model is briefly described, and a new positioning algorithm is announced. Obtaining the solution from the non-linear system of equations is not trivial as presented by numerous publications. One of the most important problems still to be solved in the 21st century is the problem of non-linearity of equations. Observations are linearized to solve the problem of positioning – such as the distances or the pseudoranges. Moreover, most positioning solutions are based on numerical computations. In this article the basic principles of the methods for solving the positioning problem are presented, and the formulas and their derivation are given. The numerical example with simulated data and proof confirm the correct performance of the proposed algorithm. A new algorithm for determining of the point coordinates and a systematic error in two-dimensional space in geodetic network solution is presented. In the proposed solution there is no need to know the initial approximate location of the determined point, nor the coordinates of the transition points.

1. INTRODUCTION

Generally, most of the observed phenomena in technical sciences such as geodesy, and navigation, and positioning are described by non-linear equations. Surveying, navigation and positioning seems to be nothing extraordinary but a very or even the most important part of the mathematical model. Obtaining the solution from the non-linear system of equations is not trivial as presented by numerous publications. In Leva [1] solution there is a system of two linear equations combined together with a range difference and equations equivalent to the system of pseudorange equations. This formulation represents the user's position as the intersection of two planes and a hyperbola branch of revolution. Krause [2] introduced a two step algorithm for the direct positioning solution. In the first step the receiver clock offset $c\ dt$ is determined involving the inversion of $(2x2)$ matrix. In the second step, vectors from the satellites to the receiver can be evaluated and through these vectors addition, the receiver position is computed. The vectorial approach is also evidenced in the works of Abel and Chafee [3,4], Hofmann-Wellenhof et al. [5]. Subsequent options for determining a unique solution among many others was discussed by Kleusberg [6] and Caravantes et al. [7]. Many authors analysed the pseudoranging four point problem (known as "pseudo 4P" problem). In the abovementioned algorithm four pseudorange equations and geometrical conditions of various cases were analysed. Solution given by Grafarend and Chan [8] is based on the quadratic form of four pseudorange system of equations for three stage algebraic reduction of numbers of observation equations. In their solution, at the end of the third computation stage, there is

a quadratic equation and there is a problem to identify the point where the solution of the quadratic equation bifurcates (this problem is known as the bifurcation problem). Positioning algorithms mostly produces two distinct solutions, in which case additional constrains are used to determine the correct solution. However, some elegant algebraic manipulation was published by Bancroft [9] to reduce the equations to avoid a least-squares method for four observed pseudoranges. Thus for 4 observed satellites there is a direct solution for a position calculation. Bancroft's algorithms involves the inversion of a $(4x4)$ matrix and the solution of a scalar equation of second order. In fact, for more than 4 satellites the least-squares solution solves the normal equation. Normal equations are often used in surveying and navigation for non-square matrices to solve an inverse matrix. Awange and Grafarend [10] presented an algebraic solution using multipolynomial resultant and Groebner basis. Its direct application is the elimination of variables in nonlinear systems of equations.
The author has provided another new algorithm for the determination of the coordinates and a systematic error without the need of the linearization technique and least squares method. The least squares method can be used but it is not mandatory to solve a positioning problem.

2. MATHEMATICAL MODEL AND NUMERICAL EXAMPLE OF A NEW POSITIONING ALGORITHM.

2.1 DETERMINATION OF THE COORDINATES OF A POINT $Q(x_Q, y_Q)$ AND A SYSTEMATIC ERROR δ_Q IN 2D ON THE BASIS OF REFERENCE AND TRANSITION POINTS

The example of determination of the coordinates (Fig. 1) of a point $Q(x_Q, y_Q)$ and a systematic error δ_Q on the basis of known network coordinates of four reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ is given.
There are known four distances $d_{aQ}, d_{bQ}, d_{cQ}, d_{dQ}$ from the unknown transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$ to point $Q(x_Q, y_Q)$.
There are also known distances:
$d_{1a}, d_{1b}, d_{1c}, d_{1d}, d_{2a}, d_{2b}, d_{2c}, d_{2d}, d_{3a}, d_{3b}, d_{3c}, d_{3d}, d_{4a}, d_{4b}, d_{4c}, d_{4d}$ between four reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ and the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$.

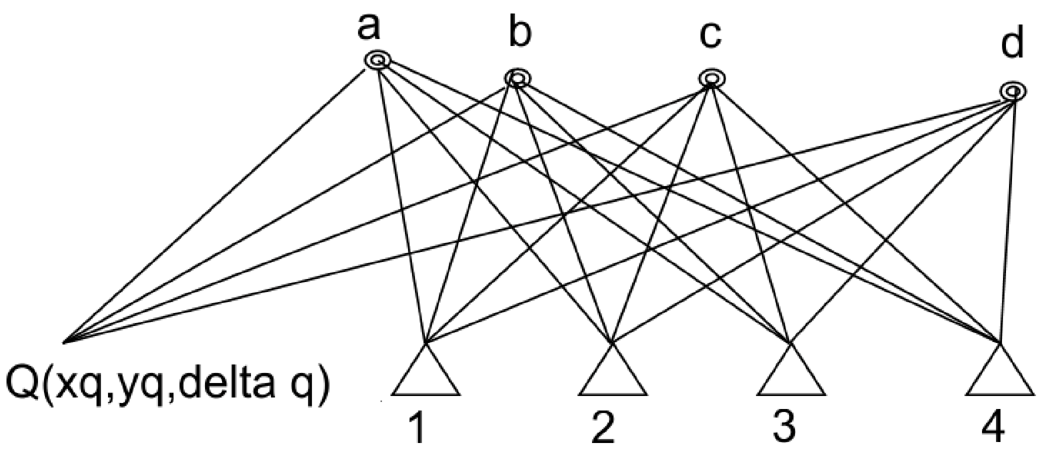

Fig.1. The network of the reference and transition points

The coordinates of point $Q(x_Q, y_Q)$ and a systematic error δ_Q can be computed from the following author's formula [13]:

$$[x_Q \quad y_Q \quad \delta_Q] = \frac{1}{2}[\,T_{ab} \quad T_{ac} \quad T_{ad}] \begin{bmatrix} \Delta X_{ab} & \Delta Y_{ab} & d_{aQ} - d_{bQ} \\ \Delta X_{ac} & \Delta Y_{ac} & d_{aQ} - d_{cQ} \\ \Delta X_{ad} & \Delta Y_{ad} & d_{aQ} - d_{dQ} \end{bmatrix}^{-1} \quad (1)$$

$T_{ab} -$ difference of two transition point indicators t_{bQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,
$T_{ac} -$ difference of two transition point indicators t_{cQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,
$T_{ad} -$ difference of two transition point indicators t_{dQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,

The simulated input data of the numerical example are as follows:

$$x_1 = 700\,m; y_1 = 800\,m; x_2 = 300\,m; y_2 = 600\,m;$$
$$x_3 = 800\,m; y_3 = 250\,m; x_4 = 900\,m; y_4 = 300\,m;$$
$$d_{1a} = 640{,}3124\,m;\ d_{1b} = 254{,}2833\,m;\ d_{1c} = 901{,}2547\,m; d_{1d} = 257{,}4102\,m;$$
$$d_{2a} = 700{,}0000\,m;\ d_{2b} = 597{,}2102\,m;\ d_{2c} = 1258{,}7000\,m; d_{2d} = 195{,}6016\,m;$$
$$d_{3a} = 1163{,}0000\,m;\ d_{3b} = 811{,}7635\,m;\ d_{3c} = 1416{,}5000\,m;\ d_{3d} = 510{,}4508\,m;$$
$$d_{4a} = 1166{,}2000\,m;\ d_{4b} = 783{,}2369\,m;\ d_{4c} = 1356{,}9000\,m;\ d_{4d} = 543{,}1943\,m;$$
$$d_{aQ} = 1216{,}6000\,m, d_{bQ} = 926{,}4232\,m, d_{cQ} = 534{,}6588\,m; d_{dQ} = 1319{,}0000\,m;$$

Values of the indicators t_{1a} ; t_{2a} ; t_{3a} can be found on the basis of a fixed point indicator definition given by S. Hausbrandt [11], this was also published in cracovian form invented by Banachiewicz [12], and they are computed to the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$:

$$t_{1a} = 720000; t_{2a} = -40000; t_{3a} = -650069; t_{4a} = 460002;$$

The differences between reference point indicators [13] $\Delta t_{12_a}, \Delta t_{13_a}, \Delta t_{14_a}, \Delta t_{12_b}, \Delta t_{13_b}, \Delta t_{14_b}, \Delta t_{12_c}, \Delta t_{13_c}, \Delta t_{14_c}, \Delta t_{12_d}, \Delta t_{13_d}, \Delta t_{14_d}$ are computed in regard to the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$:

$$\Delta t_{12_a} = -760000; , \Delta t_{13_a} = -1370000;\ , \Delta t_{14_a} = -1180000;$$
$$\Delta t_{12_b} = -972000;\ , \Delta t_{13_b} = -1021800; \Delta t_{14_b} = -778800;$$
$$\Delta t_{12_c} = -1452000; , \Delta t_{13_c} = -1621800;\ , \Delta t_{14_c} = -1258800;$$
$$\Delta t_{12_d} = -652000;\ , \Delta t_{13_d} = -621800; \Delta t_{14_d} = -458800;$$

Values of matrices: $K_a^{-1}, K_b^{-1}, K_c^{-1}, K_d^{-1}$ can be computed:

$$K_a = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{a21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{a31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{a41} \end{bmatrix}^{-1} = \begin{bmatrix} K_{a1,1} & K_{a1,2} & K_{a1,3} \\ K_{a2,1} & K_{a2,2} & K_{a2,3} \\ K_{a3,1} & K_{a3,2} & K_{a3,3} \end{bmatrix} =$$
$$= \begin{bmatrix} -0{,}0019 & -0{,}0005 & 0{,}0003 \\ -0{,}0007 & -0{,}0013 & -0{,}0014 \\ 0{,}0015 & -0{,}0004 & 0{,}0014 \end{bmatrix};$$
$$K^{-1}{}_b = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{b21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{b31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{b41} \end{bmatrix}^{-1} = \begin{bmatrix} K_{b1,1} & K_{b1,2} & K_{b1,3} \\ K_{b2,1} & K_{b2,2} & K_{b2,3} \\ K_{b3,1} & K_{b3,2} & K_{b3,3} \end{bmatrix} =$$
$$= \begin{bmatrix} -0{,}0009 & 0{,}0043 & -0{,}0044 \\ -0{,}0048 & -0{,}0204 & 0{,}0175 \\ 0{,}0056 & 0{,}0187 & -0{,}0175 \end{bmatrix};$$
$$K^{-1}{}_c = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{c21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{c31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{c41} \end{bmatrix}^{-1} = \begin{bmatrix} K_{c1,1} & K_{c1,2} & K_{c1,3} \\ K_{c2,1} & K_{c2,2} & K_{c2,3} \\ K_{c3,1} & K_{c3,2} & K_{c3,3} \end{bmatrix} =$$
$$= \begin{bmatrix} 0{,}0010 & 0{,}0081 & -0{,}0084 \\ -0{,}0123 & -0{,}0356 & 0{,}0337 \\ 0{,}0131 & 0{,}0340 & -0{,}0337 \end{bmatrix};$$
$$K^{-1}{}_d = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{d21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{d31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{d41} \end{bmatrix}^{-1} = \begin{bmatrix} K_{d1,1} & K_{d1,2} & K_{d1,3} \\ K_{d2,1} & K_{d2,2} & K_{d2,3} \\ K_{d3,1} & K_{d3,2} & K_{d3,3} \end{bmatrix} =$$
$$= \begin{bmatrix} -0{,}0074 & 0{,}0053 & -0{,}0145 \\ 0{,}0212 & -0{,}0246 & 0{,}0578 \\ -0{,}0204 & 0{,}0229 & -0{,}0578 \end{bmatrix};$$

Values of the transition point indicators $t_{aQ}, t_{bQ}, t_{cQ}, t_{dQ}$ can be computed [13]:

$$t_{aQ} = \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * \right.$$
$$\left. * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 - \mathrm{d}_{\mathrm{aQ}}^2 = 300000;$$

$$t_{bQ} = \left(\tfrac{1}{2}\Delta t_{12_b} * K_{b1,1} + \tfrac{1}{2}\Delta t_{13_b} * K_{b2,1} + \tfrac{1}{2}\Delta t_{14_b} * K_{b3,1}\right)^2 + \left(\tfrac{1}{2}\Delta t_{12_b} * K_{b1,2} + \tfrac{1}{2}\Delta t_{13_b} * * K_{b2,2} + \tfrac{1}{2}\Delta t_{14_b} * K_{b3,2}\right)^2 - \mathrm{d}_{\mathrm{bQ}}^2 = 726000;$$

$$t_{cQ} = \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,1} + \tfrac{1}{2}\Delta t_{13_c} * K_{c2,1} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,1}\right)^2 + \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,2} + \tfrac{1}{2}\Delta t_{13_c} * * K_{c2,2} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,2}\right)^2 - \mathrm{d}_{\mathrm{cQ}}^2 = 3426000;$$

$$t_{dQ} = \left(\tfrac{1}{2}\Delta t_{12_d} * K_{d1,1} + \tfrac{1}{2}\Delta t_{13_d} * K_{d2,1} + \tfrac{1}{2}\Delta t_{14_d} * K_{d3,1}\right)^2 + \left(\tfrac{1}{2}\Delta t_{12_d} * K_{d1,2} + \tfrac{1}{2}\Delta t_{13_d} * * K_{d2,2} + \tfrac{1}{2}\Delta t_{14_d} * K_{d3,2}\right)^2 - \mathrm{d}_{\mathrm{dQ}}^2 = -1074000;$$

The differences of transition point indicators T_{ab}, T_{ac}, T_{ad} can be computed in regard to point $Q(x_Q, y_Q)$:

$$T_{ab} = t_{bQ} - t_{aQ} = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right)^2 + + \left(\tfrac{1}{2}\Delta t_{12_b} * K_{b1,2} + \tfrac{1}{2}\Delta t_{13_b} * K_{b2,2} + \tfrac{1}{2}\Delta t_{14_b} * K_{b3,2}\right)^2 - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,2} + \tfrac{1}{2}\Delta t_{13_a} * * K_{a2,2} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 - \mathrm{d}_{\mathrm{bQ}}^2 = 426000;$$

$$T_{ac} = t_{cQ} - t_{aQ} = \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,1} + \tfrac{1}{2}\Delta t_{13_c} * K_{c2,1} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,1}\right)^2 + \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,2} + + \tfrac{1}{2}\Delta t_{13_c} * K_{c2,2} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,2}\right)^2 - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,1} + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,1} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,1}\right)^2 + - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,2} + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,2} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 - \mathrm{d}_{\mathrm{cQ}}^2 = = 3126000$$

$$\begin{aligned} T_{ad} = t_{dQ} - t_{aQ} = \\ &= \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1} +\right)^2 \\ &+ \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * K_{d2,2} + + \frac{1}{2}\Delta t_{14_d} * K_{d3,2} +\right)^2 \\ &- \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + + \frac{1}{2}\Delta t_{14_a} * K_{a3,1} +\right)^2 \\ &- \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} ** K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 - \mathrm{d}_{\mathrm{dQ}}^2 = \\ &= -1374000 \end{aligned}$$

Partial coordinate differences $X_{ab}, Y_{ab}, X_{ac}, Y_{ac}, X_{ad}, Y_{ad}$ of the unknown transition points $a(x_a, y_a)$, $b(x_b, y_b)$, $c(x_c, y_c)$, $d(x_d, y_d)$ can be defined as:

$$X_{ab} = x_b - x_a = \left(\tfrac{1}{2}\Delta t_{12_b} * K_{b1,1} + \tfrac{1}{2}\Delta t_{13_b} * K_{b2,1} + \tfrac{1}{2}\Delta t_{14_b} * K_{b3,1}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,1} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,1}\right) = 388;$$
$$Y_{ab} = y_b - y_a = \left(\tfrac{1}{2}\Delta t_{12_b} * K_{b1,2} + \tfrac{1}{2}\Delta t_{13_b} * K_{b2,2} + \tfrac{1}{2}\Delta t_{14_b} * K_{b3,2}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,2} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,2}\right) = -246;$$
$$X_{ac} = x_c - x_a = \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,1} + \tfrac{1}{2}\Delta t_{13_c} * K_{c2,1} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,1}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,1} + + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,1}\right) = 688;$$
$$Y_{ac} = y_c - y_a = \left(\tfrac{1}{2}\Delta t_{12_c} * K_{c1,2} + \tfrac{1}{2}\Delta t_{13_c} * K_{c2,2} + \tfrac{1}{2}\Delta t_{14_c} * K_{c3,2}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,2} + + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,2}\right) = 354;$$
$$X_{ad} = x_d - x_a = \left(\tfrac{1}{2}\Delta t_{12_d} * K_{d1,1} + \tfrac{1}{2}\Delta t_{13_d} * K_{d2,1} + \tfrac{1}{2}\Delta t_{14_d} * K_{d3,1}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,1} + + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,1}\right) = 188;$$
$$Y_{ad} = y_d - y_a = \left(\tfrac{1}{2}\Delta t_{12_d} * K_{d1,2} + \tfrac{1}{2}\Delta t_{13_d} * K_{d2,2} + \tfrac{1}{2}\Delta t_{14_d} * K_{d3,2}\right) - \left(\tfrac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \tfrac{1}{2}\Delta t_{13_a} * K_{a2,2} + \tfrac{1}{2}\Delta t_{14_a} * K_{a3,2}\right) = -646;$$

According to the formula (1) the coordinates of point $Q(x_Q, y_Q)$ and a systematic error δ_Q can be computed:

$$[x_Q \quad y_Q \quad \delta_Q] = \frac{1}{2}[T_{ab} \quad T_{ac} \quad T_{ad}] \begin{bmatrix} \Delta X_{ab} & \Delta Y_{ab} & \Delta d_{1Q} \\ \Delta X_{ac} & \Delta Y_{ac} & \Delta d_{2Q} \\ \Delta X_{ad} & \Delta Y_{ad} & \Delta d_{3Q} \end{bmatrix}^{-1}$$
$$x_Q = 1500\ m; y_Q = 1500\ m;\ \delta_Q = 3{,}8199 * 10^{-11}\ m;$$

For the result confirmation, the geometric distances from $Q(x_Q, y_Q)$ to the transition points can be computed.

2.2 PROOF

The coordinate equations for the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$ and unknown systematic errors $\delta_a, \delta_b, \delta_c, \delta_d$ in observations: $d_{1a}, d_{1b}, d_{1c}, d_{1d}, d_{2a}, d_{2b}, d_{2c}, d_{2d}, d_{3a}, d_{3b}, d_{3c}, d_{3d}, d_{4a}, d_{4b}, d_{4c}, d_{4d}$, respectively, can be expressed as follows:

$$[x_a \quad y_a \quad \delta_a] = \tfrac{1}{2}[\Delta t_{12_a} \quad \Delta t_{13_a} \quad \Delta t_{14_a}] * K_a^{-1}; \tag{2}$$
$$[x_b \quad y_b \quad \delta_b] = \frac{1}{2}[\Delta t_{12_b} \quad \Delta t_{13_b} \quad \Delta t_{14_b}] * K_b^{-1};$$
$$[x_c \quad y_c \quad \delta_c] = \frac{1}{2}[\Delta t_{12_c} \quad \Delta t_{13_c} \quad \Delta t_{14_c}] * K_c^{-1};$$
$$[x_d \quad y_d \quad \delta_d] = \frac{1}{2}[\Delta t_{12_d} \quad \Delta t_{13_d} \quad \Delta t_{14_d}] * K_d^{-1};$$

where:

Matrices $K_a^{-1}, K_b^{-1}, K_c^{-1}, K_d^{-1}$ can be defined by using the partial coordinates to the reference point coordinates $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ and distance differences from the reference points do the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$. Thus abovementioned matrices can be expressed as follows:

$$K_a^{-1} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{a21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{a31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{a41} \end{bmatrix}^{-1} ;$$

Δx_{12} – the partial coordinates x_1 and x_2 of the reference points $1(x_1, y_1), 2(x_2, y_2)$
Δx_{13} – the partial coordinates x_1 and x_3 of the reference points $1(x_1, y_1), 3(x_3, y_3)$
Δx_{14} – the partial coordinates x_1 and x_4 of the reference points $1(x_1, y_1), 4(x_4, y_4)$
Δy_{12} – the partial coordinates y_1 and y_2 of the reference points $1(x_1, y_1), 2(x_2, y_2)$
Δy_{13} – the partial coordinates y_1 and y_3 of the reference points $1(x_1, y_1), 3(x_3, y_3)$
Δy_{14} – the partial coordinates y_1 and y_4 of the reference points $1(x_1, y_1), 4(x_4, y_4)$

Δd_{a21} – distance difference from the reference points $1(x_1, y_1), 2(x_2, y_2)$ to transition point $a(x_a, y_a)$
Δd_{a31} – distance difference from the reference points $3(x_3, y_3)$,$1(x_1, y_1)$ to transition point $a(x_a, y_a)$
Δd_{a41} – distance difference from the reference points $4(x_4, y_4), 1(x_1, y_1)$ to transition point $a(x_a, y_a)$

Values of matrix K_a^{-1} are as follows:

$$K_{a1,1} = \frac{\Delta x_{12}}{det(K_a)}; K_{a2,1} = \frac{\Delta x_{13}}{det(K_a)}; K_{a3,1} = \frac{\Delta x_{14}}{det(K_a)};$$
$$K_{a1,2} = \frac{\Delta y_{12}}{det(K_a)}; K_{a2,2} = \frac{\Delta y_{13}}{det(K_a)}; K_{a3,2} = \frac{\Delta y_{14}}{det(K_a)};$$
$$K_{a1,3} = \frac{\Delta d_{a21}}{det(K_a)}; K_{a2,3} = \frac{\Delta d_{a31}}{det(K_a)}; K_{a3,3} = \frac{\Delta d_{a41}}{det(K_a)};$$

Matrix K_b^{-1}:

$$K_b^{-1} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{b21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{b31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{b41} \end{bmatrix}^{-1} ;$$

Δd_{b21} – distance difference from the reference points $1(x_1, y_1), 2(x_2, y_2)$ to transition point $b(x_b, y_b)$
Δd_{b31} – distance difference from the reference points $3(x_3, y_3)$,$1(x_1, y_1)$ to transition point $b(x_b, y_b)$
Δd_{b41} – distance difference from the reference points $4(x_4, y_4)$,$1(x_1, y_1)$ to transition point $b(x_b, y_b)$

Values of matrix K_b^{-1} are as follows:

$$K_{b1,1} = \frac{\Delta x_{12}}{det(K_b)};\ K_{b2,1} = \frac{\Delta x_{13}}{det(K_b)};\ K_{b3,1} = \frac{\Delta x_{14}}{det(K_b)};$$
$$K_{b1,2} = \frac{\Delta y_{12}}{det(K_b)};\ K_{b2,2} = \frac{\Delta y_{13}}{det(K_b)};\ K_{b3,2} = \frac{\Delta y_{14}}{det(K_b)};$$
$$K_{b1,3} = \frac{\Delta d_{b21}}{det(K_b)};\ K_{b2,3} = \frac{\Delta d_{b31}}{det(K_b)};\ K_{b3,3} = \frac{\Delta d_{b41}}{det(K_b)};$$

Macierz K_c^{-1}:

$$K_c^{-1} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{c21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{c31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{c41} \end{bmatrix}^{-1};$$

Δd_{c21} – distance difference from the reference points $1(x_1, y_1), 2(x_2, y_2)$ to transition point $c(x_c, y_c)$

Δd_{c31} – distance difference from the reference points $3(x_3, y_3)$,$1(x_1, y_1)$ to transition point $c(x_c, y_c)$

Δd_{c41} – distance difference from the reference points $4(x_4, y_4)$,$1(x_1, y_1)$ to transition point $c(x_c, y_c)$

Values of matrix K_c^{-1} are as follows:

$$K_{c1,1} = \frac{\Delta x_{12}}{det(K_c)};\ K_{c2,1} = \frac{\Delta x_{13}}{det(K_c)};\ K_{c3,1} = \frac{\Delta x_{14}}{det(K_c)};$$
$$K_{c1,2} = \frac{\Delta y_{12}}{det(K_c)};\ K_{c2,2} = \frac{\Delta y_{13}}{det(K_c)};\ K_{c3,2} = \frac{\Delta y_{14}}{det(K_c)};$$
$$K_{c1,3} = \frac{\Delta d_{c21}}{det(K_c)};\ K_{c2,3} = \frac{\Delta d_{c31}}{det(K_c)};\ K_{c3,3} = \frac{\Delta d_{c41}}{det(K_c)};$$

Matrix K_d^{-1}:

$$K_d^{-1} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{d21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{d31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{d41} \end{bmatrix}^{-1};$$

Δd_{d21} – distance difference from the reference points $1(x_1, y_1), 2(x_2, y_2)$ to transition point $d(x_d, y_d)$

Δd_{d31} – distance difference from the reference points $3(x_3, y_3)$,$1(x_1, y_1)$ to transition point $d(x_d, y_d)$

Δd_{d41} – distance difference from the reference points $4(x_4, y_4)$,$1(x_1, y_1)$ to transition point $d(x_d, y_d)$

Values of matrix K_d^{-1} are as follows:

$$K_{d1,1} = \frac{\Delta x_{12}}{det(K_d)};\ K_{d2,1} = \frac{\Delta x_{13}}{det(K_d)};\ K_{d3,1} = \frac{\Delta x_{14}}{det(K_d)};$$
$$K_{d1,2} = \frac{\Delta y_{12}}{det(K_d)};\ K_{d2,2} = \frac{\Delta y_{13}}{det(K_d)};\ K_{d3,2} = \frac{\Delta y_{14}}{det(K_d)};$$

$$K_{d1,3} = \frac{\Delta d_{d21}}{det(K_d)}; K_{d2,3} = \frac{\Delta d_{d31}}{det(K_d)}; K_{d3,3} = \frac{\Delta d_{d41}}{det(K_d)};$$

The coordinates of the transition points $a(x_a, y_a)$,$b(x_b, y_b)$,$c(x_c, y_c)$, $d(x_d, y_d)$ and unknown systematic errors $\delta_a, \delta_b, \delta_c, \delta_d$ respectively, can be expressed as follows:

$$x_a = \frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1} \quad (3)$$
$$y_a = \frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2};$$
$$\delta_a = \frac{1}{2}\Delta t_{12_a} * K_{a1,3} + \frac{1}{2}\Delta t_{13_a} * K_{a2,3} + \frac{1}{2}\Delta t_{14_a} * K_{a3,3};$$
$$x_b = \frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1};$$
$$y_b = \frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \frac{1}{2}\Delta t_{13_b} * K_{b2,2} + \frac{1}{2}\Delta t_{14_b} * K_{b3,2};$$
$$\delta_b = \frac{1}{2}\Delta t_{12_b} * K_{b1,3} + \frac{1}{2}\Delta t_{13_b} * K_{b2,3} + \frac{1}{2}\Delta t_{14_b} * K_{b3,3};$$
$$x_c = \frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1};$$
$$y_c = \frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \frac{1}{2}\Delta t_{13_c} * K_{c2,2} + \frac{1}{2}\Delta t_{14_c} * K_{c3,2};$$
$$\delta_c = \frac{1}{2}\Delta t_{12_c} * K_{c1,3} + \frac{1}{2}\Delta t_{13_c} * K_{c2,3} + \frac{1}{2}\Delta t_{14_c} * K_{c3,3};$$
$$x_d = \frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1};$$
$$y_d = \frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * K_{d2,2} + \frac{1}{2}\Delta t_{14_d} * K_{d3,2};$$
$$\delta_d = \frac{1}{2}\Delta t_{12_d} * K_{d1,3} + \frac{1}{2}\Delta t_{13_d} * K_{d2,3} + \frac{1}{2}\Delta t_{14_d} * K_{d3,3};$$

The transition point indicators $a(x_a, y_a)$, $b(x_b, y_b)$,$c(x_c, y_c)$, $d(x_d, y_d)$ to point $Q(x_Q, y_Q)$ can be expressed as follows:

(4)

$$t_{aQ} = \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * \right.$$
$$\left. * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 - \mathrm{d}_{\mathrm{aQ}}^2;$$
$$t_{bQ} = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \frac{1}{2}\Delta t_{13_b} * \right.$$
$$\left. * K_{b2,2} + \frac{1}{2}\Delta t_{14_b} * K_{b3,2}\right)^2 - \mathrm{d}_{\mathrm{bQ}}^2;$$
$$t_{cQ} = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \frac{1}{2}\Delta t_{13_c} * \right.$$
$$\left. * K_{c2,2} + \frac{1}{2}\Delta t_{14_c} * K_{c3,2}\right)^2 - \mathrm{d}_{\mathrm{cQ}}^2;$$
$$t_{dQ} = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * \right.$$
$$\left. K_{d2,2} + \frac{1}{2}\Delta t_{14_d} * K_{d3,2}\right)^2 - \mathrm{d}_{\mathrm{dQ}}^2;$$

First equation (t_{aQ}) was subtracted from the second equation (t_{bQ}) and t_{aQ} was subtracted from the third equation (t_{cQ}). Thus the differences of the transition point indicators T_{ab}, T_{ac}, T_{ad} to point $Q(x_Q, y_Q)$ can be calculated as follows:

(5)

$$T_{ab} = t_{bQ} - t_{aQ} = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_b} * K_{b2,2} + + \frac{1}{2}\Delta t_{14_b} * K_{b3,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 - \mathrm{d}_{\mathrm{bQ}}^2;$$
$$T_{ac} = t_{cQ} - t_{aQ} = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_c} * K_{c2,2} + + \frac{1}{2}\Delta t_{14_c} * K_{c3,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * \right.$$
$$\left. * K_{a3,1}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} ** K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 - \mathrm{d}_{\mathrm{cQ}}^2;$$
$$T_{ad} = t_{dQ} - t_{aQ} = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1}\right)^2 +$$
$$+ \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * K_{d2,2} + + \frac{1}{2}\Delta t_{14_d} * K_{d3,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * \right.$$
$$\left. * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 + \mathrm{d}_{\mathrm{aQ}}^2 +$$
$$- \mathrm{d}_{\mathrm{dQ}}^2;$$

The difference of the partial coordinates of unknown transition point coordinates $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$ can be expressed by using the reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ as follows:

(6)

$$X_{ab} = x_b - x_a = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$
$$Y_{ab} = y_b - y_a = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \frac{1}{2}\Delta t_{13_b} * K_{b2,2} + \frac{1}{2}\Delta t_{14_b} * K_{b3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$
$$X_{ac} = x_c - x_a = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$
$$Y_{ac} = y_c - y_a = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \frac{1}{2}\Delta t_{13_c} * K_{c2,2} + \frac{1}{2}\Delta t_{14_c} * K_{c3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$
$$X_{ad} = x_d - x_a = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$

$$Y_{ad} = y_d - y_a = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * K_{d2,2} + \frac{1}{2}\Delta t_{14_d} * K_{d3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$

Values T_{ab}, T_{ac}, $T_{ad}, X_{ab}, Y_{ab}, X_{ac}, Y_{ac}, X_{ad}, Y_{ad}$ were already introduced. Thus the coordinates of a point $Q(x_Q, y_Q)$ and unknown δ_Q can be computed according to the formula (1):

$$[x_Q \quad y_Q \quad \delta_Q] = \frac{1}{2}[T_{ab} \quad T_{ac} \quad T_{ad}] \begin{bmatrix} \Delta X_{ab} & \Delta Y_{ab} & \Delta d_{abQ} \\ \Delta X_{ac} & \Delta Y_{ac} & \Delta d_{acQ} \\ \Delta X_{ad} & \Delta Y_{ad} & \Delta d_{adQ} \end{bmatrix}^{-1}$$

3. CONCLUSIONS

A new algorithm for determining of the point coordinates of Q and the systematic error δ_Q in two-dimensional space in geodetic network solution was presented. Transition point indicator definition is based on the reference point indicator definitions both developed by the author [13]. With the use of definitions of these point indicators it is possible to determine the sought point Q in geodetic network solution. The direct solution for position determination of unknown point is derived without application of the least squares method. In the proposed solution there is no need to know the initial approximate location of the determined point. In the presented algorithm there is also no need to know the values of coordinates of transition points. The basic principles of the methods for solving the positioning problem in geodetic networks, the formulas and their derivation has been presented. The numerical example with simulated data and a proof confirm the correct performance of the proposed algorithm.

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BIOGRAPHICAL NOTES

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