Refraction Coefficient Determination And Modelling For The Territory Of The Kingdom Of Saudi Arabia

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Key words: refraction coefficient, precise levelling, Kingdom of Saudi Arabia, refraction coefficient modelling

SUMMARY

The current paper presents the results from the Refraction Coefficient Determination for Precise Levelling Observation (RCD_PLO) project in relation to the establishment of a new National Vertical Reference Frame for the Kingdom of Saudi Arabia. A specialised software for processing more than 1 200 000 records of precise levelling measurements and temperature observation data was developed and used for refraction coefficient modelling. The modelling was based on the Kukkamaki's classical formula and a newly derived formula for the refraction coefficient, taking into account the topography roughness along the line of sight by employing the so-called 'equivalent heights'. The results were subjected to statistical and correlation analyses and later validated using levelling line and loop misclosures. 2D and 3D (GIS-based) models of refraction coefficient were also build. At the end of the paper, recommendations for the usage of the derived refraction coefficient models are given, considering the required levelling accuracy, height levels, type of the atmosphere (normal or inverse) and availability of temperature measurements.

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1. INTRODUCTION

Precise levelling plays an essential role in establishing a National Vertical Reference System (NVRS). The high accuracy of precise networks ensures the required accuracy for the lower order networks, which are directly used for many applications related to geodesy and surveying. It is affected, however, by many factors; one of them is vertical refraction. Extensive research shows that the influence of vertical refraction on the line of sight during geodetic activities and particularly precise levelling depends on the topography roughness along the levelling line and the air temperature. The refraction effect could reach up to 1-2 mm (with an accuracy estimate of 0.2-0.6 mm) on a measured height difference per setup and thus it could increase the final value of the loop misclosure. The software incorporated in present-day levelling instruments could provide a refraction correction based on a standard atmospheric model for air pressure, temperature and humidity. However, in most cases the standard model is insufficient considering that precise levelling is carried out in various atmospheric conditions and/or rough topography. On the other hand, if precise temperature observations obtained during levelling are available, the refraction effect could be modelled and used for correcting the acquired measurements. Consequently, this would improve the accuracy of the levelling networks.

The aim of this paper is to present the results from the Refraction Coefficient Determination for Precise Levelling Observation (RCD_PLO) project related to the establishment of a new National Vertical Reference Frame for the Kingdom of Saudi Arabia (KSA). The refraction is modelled via the temperature triplets collected during the precise levelling, taking into account the topography roughness along the line of sight by employing the so-called ‘equivalent height’. Considering the amount of data to be processed (more than 1 200 000 records), a specialised software was developed. As a result, refraction coefficients per levelling section were obtained. For assessing the relevant accuracy statistical analysis and correlation analysis were used. In addition, comparison was also made with results based on Kukkamaki’s classical formula for refraction. As project deliverables, several different refraction coefficients values were produced. Furthermore, the results were visualised using 2D and 3D (GIS-based) modelling and validated with respect to the relevant levelling line and loop misclosures. The latter have dropped significantly as compared to computed misclosures without accounting for the effects of refraction. At the end of this paper, some recommendations for application of the derived refraction coefficients models are presented.

2. PROBLEM BACKGROUND

2.1 Past research on refraction in geodetic levelling

The influence of refraction on precise geodetic levelling is discussed in many studies among which are Kukkamaki (1938, 1939), Holdahl (1979), Whalen (1981), Agnus-Leppan (1984), Heer and Niemier (1985), Nakahori and Kanno (1985), Stein et al. (1986), Castle et al. (1994) and others. Most of them comment on the application of the Kukkamaki's refraction equation (1938) but acknowledge the refraction dependence from detailed temperature profile at the setup and topography roughness (Heer and Niemier, 1985; Agnus-Leppan, 1984, Whalen, 1981); Островский, А. Л. и др. (1990). It is important, therefore, this dependence to be accounted for during precise levelling activities conducted on the territory of the KSA, because of its flat and mountainous areas of topography with elevation range from 0 m to 2700 m and daily temperature amplitudes between 10°C and 20°C.

2.2 Refraction modelling for the KSA's precise levelling

The detailed algorithm designed for the refraction modelling over the territory of the KSA is lengthy and will take several pages and thus only its main steps will be outlined here.

The determination of vertical refraction effects on precise levelling in KSA is based on temperature measurements acquired on different reference levels above the ground. The aforementioned measurements are obtained during the last re-levelling of the KSA's National Geodetic Levelling Network (NGLN).

The measured height differences at each setup could be corrected by including a correction for vertical refraction (***C**_{ref}*) using modified Kukkamaki's formula (Kukkamaki, 1938; Kukkamaki, 1939):

(1) $$C_{ref} = d\,\frac{S^2}{(Z_0 - Z_1)^2}\,\frac{\Delta t}{\left(z_2^C - z_1^C\right)}\left[\frac{1}{C+1}\left(Z_1^{C+1} - Z_2^{C+1}\right) - Z_0^C\left(Z_1 - Z_2\right)\right],$$

where S represents the distance (in metres) between the levelling instrument and the levelling rod; Z_i (for i = 1,2) are forward and backward rod readings (in metres); Z_0 is the instrument's height (in metres); z_1 and z_2 are the reference levels of temperature readings (Fig. 1); d shows the change in the refractive index for a change of 1° in the temperature.

The last notation in (1) is $\boldsymbol{C}$, i.e. the mean refraction coefficient ($\boldsymbol{C}$-value) computed by:

(2) $$C = \ln\left(\frac{\Delta t_2}{\Delta t_1}\right)\Big/\ln(3)$$

with $\Delta t_2 = t_3 - t_2$ and $\Delta t_1 = t_2 - t_1$. The denominator in Eq. (2) is in fact the ratio between the lower temperature reference levels, namely z_1 = 0.5 m and z_2 = 1.5 m.

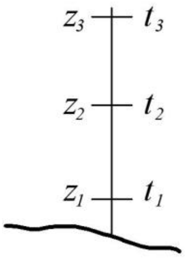


Figure 1: Reference levels z_i of temperature observations t_i

For the territory of the KSA, some of the levelling data were corrected using different ***C***-values by the Contractors performing the precise levelling.

Alternatively, the Kukkamaki's formula for refraction correction to a rod reading, as presented in (Whalen, 1981):

$$R_i = \left(ctg^2\theta\right)d\,\frac{\Delta t}{z_2^C - z_1^C}\left(\frac{1}{C+1}Z_i^{C+1} - Z_0^C Z_i + \frac{C}{C+1}Z_0^{C+1}\right), \tag{3}$$

considers the topography by including a term for the ground slope from the instrument setup towards the levelling rod. The rod readings, on the other hand, are realised on various heights above the ground and thus proving different conditions for curving of the line of sight. The influence of the topography below the line of sight could be considered more accurately by using a so-called 'equivalent height'. Its physical meaning is visually represented with the shaded (hatched) area in Fig. 2; the equivalent height h_e itself is equal to the instrument height Z_0 at the setup and to the rod reading at the levelling rod distance.

The computation of the reciprocal value of h_e is then (with notations according to Fig. 2):

$$\frac{1}{h_e} = \frac{2}{S^2}\int_0^S \frac{l}{h}\,dl = \frac{2}{S^2}\sum_{i=0}^{S} \frac{l_i}{h_i}\,\frac{l_{i+1} - l_{i-1}}{2}\ . \tag{4}$$

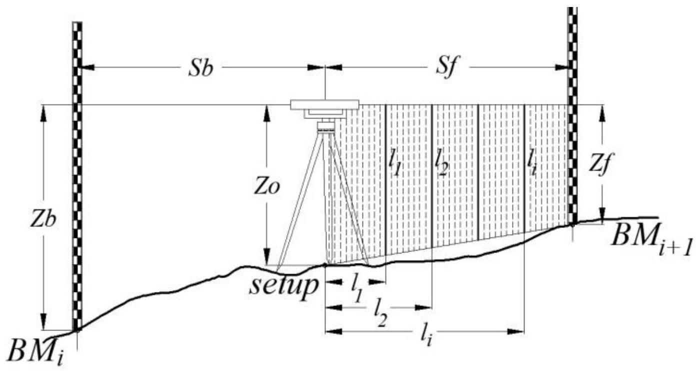


Figure 2: Representation of the equivalent height h_e for the forward rod reading

Accordingly, computing h_e requires no other variables than rod readings (backward: Z_b, and forward: Z_f) and the height of the level instrument (Z_0). In case that the latter is not measured, an average value of 1.5 – 1.6 m could be used. If the value of the h_e is large, i.e. the line of sight passes high above the ground, the influence of the terrain features on refraction will be small and vice versa.

In accordance with Eq. (1):

(5) $$C_{ref} = R_{back} - R_{for}$$

The RCD_PLO Project relies on modified version of Eq. (5).

2.3 Influence of air temperature on refraction

The effect of the vertical refraction is carried onto the measured height difference using C_{ref} or on the rod reading R_i. On the other hand, topography features along the levelling line as well as local changes in the atmosphere (wind, humidity, etc.) influence the vertical refraction itself.

The influence of the air temperature is represented by the change of the vertical temperature gradient, which has a normal (γ_a) and an abnormal part (γ_c):

(6) $$\frac{dT}{dz} = \gamma_a + \gamma_c ,$$

where: T – temperature; z – temperature's reference height of measurement.

Within the low atmospheric levels (10 m ≤ z ≤ 50 m) $\gamma_a = const = -0.0098\,°Cm^{-1}$, while γ_c strongly depends on the reference height z and is usually expressed as a function of it. In meteorology, the distribution of the temperature with height is known to follow the logarithmic relation:

(7) $$t = a + bz^c ,$$

where a, b and c are constants and z denotes the height above the ground level.

The theory of vertical refraction, on the other hand, uses three models for γ_c determination (Островский и др., 1990; Webb, 1969):

– model of a neutral atmosphere:

(8) $$\frac{dT}{dz} = const;\ \text{with}\ \gamma_a = -0.0098\,°Cm^{-1};$$

– model of an unstable atmosphere:

(9) $$\frac{dT}{dz} = cz^{-4/3} + \gamma_a;$$ with c being an inverse temperature gradient at height of z = 1 m;

– model of a stable atmosphere with an inverse temperature gradient

(10) $$\frac{dT}{dz} = cz^{-2/3} + \gamma_a;\ \text{with}\ c = -0.0098\,°Cm^{-1}$$

Since $\frac{dT}{dz}$ changes around-the-clock, its values could be positive, negative or zero. Consequently, same conclusion applied to refraction itself.

2.4 Refraction coefficient modelling

2.4.1 Refraction coefficient used in the software

The usage of Eq. (2) for refraction coefficient computation assumes fixed ratio between the temperature reference levels, equal to 3. However, some of the RCD_PLO project data have a ratio of 4.3 (see Table 1). Therefore, it was decided to model the refraction coefficient using a version of Eq. (2), taking into account the different ratio. This modification of Eq. (2) was implemented in the developed software for refraction modelling (see Sec.3.2).

2.4.2 New refraction coefficient formula

Historically, the application of the Kukkamaki's formula has not been accompanied with any specific requirements related to the reference levels of the temperature measurements. Thus, the Contractors, responsible for the levelling activities in the KSA, have implemented Eq. (2) in the data processing disregarding the inconsistency of the different temperature reference level ratios.

In order to examine the aforementioned inconsistency, a new refraction coefficient formula has been derived. The solution is based on the relation between temperature readings and their reference levels:

$$\left| \begin{array}{l} t_1 = a + b z_1^C \\ t_2 = a + b z_2^C \\ t_3 = a + b z_3^C \end{array} \right. \tag{11}$$

By forming the ratio:

$$T = \frac{t_3 - t_2}{t_2 - t_1} = \frac{\left(\frac{z_3^C}{z_1^C} - \frac{z_2^C}{z_1^C} \right)}{\left(\frac{z_2^C}{z_1^C} - 1 \right)} \tag{12}$$

for $z_1 = 0.5$ m, $z_2 = 1.5$ m and $z_3 = 2.5$ m. After some modifications, Eq. (13) below is derived. Solving it for u will lead to a solution for $\boldsymbol{C}$:

(13) $u^2 - Tu - T = 0$, where $3^{0.5C} = u$.

The final solution for $\boldsymbol{C}$ for normal and inverse atmosphere is given in Eq. (14):

$$C = 2 \frac{\log \left[\frac{T + \sqrt{T^2 + 4T}}{2} \right]}{\log 3} \tag{14}$$

3. FIELD TESTS CARRIED OUT IN THE KSA

3.1 National Vertical Network of the KSA and available data

The General Commission for Survey (GCS) of the Kingdom of Saudi Arabia (KSA) aimed to carry out the establishment of a new National Vertical Network (NVN) for the KSA. The existing NGLN of the KSA is more than 40 years old, and the majority of the network has been destroyed during the rapid infrastructure development.

Since 2010, GCS has carried out four levelling phases: Project 1, 2, 3 and 4 (see Fig. 3).

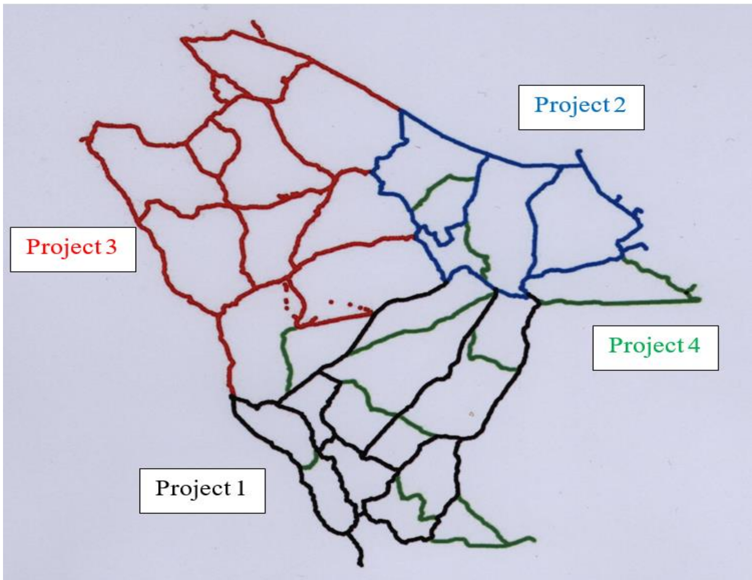


Figure 3: Location of the different NVN Projects within the KSA territory

The NVN is connected to Primary Geodetic Network (PGN) and the KSA's Tide Gauge Network. The NVN consists of levelling lines starting and ending at existing or newly-build Junction Points (JPs). The lines are connected by the JPs as depicted in Fig. 4. The JP numbering is consecutive and unique for the entire levelling network. Each JP (for example JP45, shown in Fig. 4) consists of three BMs linked together and connected to the nearest BM from the corresponding levelling line. A levelling line is defined by an initial benchmark (BM79-001) and ending benchmark (BM79-038) and consists of levelling sections (BM79-001 – BM79-002) with average length of 4 – 5 km. Between two adjacent benchmarks (BM79-001 and BM79-002) there are 4-5 interim (temporary) benchmarks with average distance of 1.2-1.5 km. Assuming an average length for line of sight of 40 m, each section is measured using 15 to 20 instrument setups. The setup numbering (1, 2, 3, etc.) is consecutive and unique only within a section. The benchmarks marking the start and the end of each section are either permanent (numbered for example BM79-001, BM79-002, etc.) or temporary (with numbering, containing letters: BM79-001A, BM79-001B, BM79-001C, etc.). Both the permanent and the temporary benchmarks have coordinates (B,L) in the WGS84 system obtained using GPS navigation during the course of the levelling. These sets of coordinates are later used for determination of approximate coordinates for each setup.

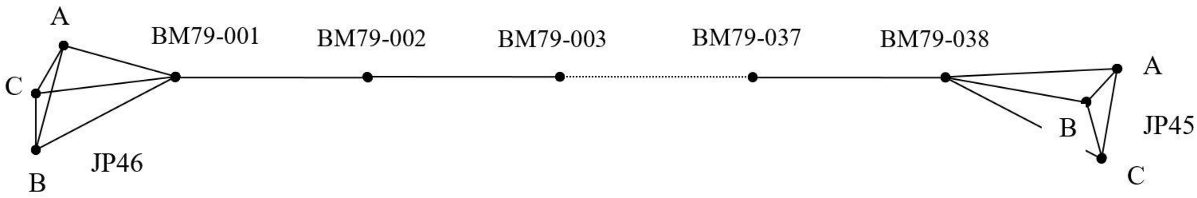


Figure 4: Junction Point (JP) connections of the Line 79 (example from Project 4)

All levelling sections of a certain levelling line are measured both in forward and backward direction. In addition to the precise levelling, at most setups simultaneous measurements of temperature at different reference levels above the ground were taken. Tables 1 and 2 give some main characteristics of the four levelling Projects.

Table 1: Characteristics of the four levelling Projects

Pr. No	Elevation [m]		Temperature [°C]			Temperature reference level [m]			***C***-value used by the Contractors	Measurement period (month, year)
	Low	High	Low	High	Ave	z_1	z_2	z_3		
1	2	1600	6	47	25.6	0.3	1.3	---	-0.347	VI – VIII.12
						0.5	1.5	2.5		IX – III.13
2	1	783	3.1	47	24.8	0.3	1.3	---	-0.347	III – IX.12
						0.5	1.5	2.5		
3	3	1060	6	47	27.0	0.5	1.5	2.5	-0.347	IX.13 – III.14
4	2	2097	-0.7	46	29.5	0.5	1.5	2.5	-0.410	IX.15 – III.16

Table 2: Amount of data from the NVN (all Projects) to be processed

Number of lines	80
Number of sections in lines	13 250
Number of setups	> 310 000
Total length in km	> 15 000
Total number of benchmarks	3 334
Number of new benchmarks	2 720
Number of recorded levelling data (both backward and forward direction)	> 620 000
Number of temperature records	> 580 000

3.2 Software development

The amount of data to be processed for refraction coefficient estimations enforced the development of a specialised software package: REFRACTION. The programme language used is FORTRAN 90 on a Windows platform. A brief summary and a flowchart of REFRACTION modules are presented in Table 3 and Figure 5.

Table 3: Function of the different REFRACTION submodules

1. Convert	1. Converts source files **.lvl* into working **.txt* files for each line 2. Creates ***_Aver.txt*** files for each section
1.1 Con_P12	Converts the original files containing the temperature observations related to the Projects 1 and 2 into the **.txt* files
1.2 Con_P34	Converts the original files containing the temperature observations related to the Projects 3 and 4 into the ***.txt*** files
2. Benchmarks	Reads the (B,L,H) coordinates and heights of the permanent benchmarks
3. Hight_Benchmarks	Computes the height H of the temporary benchmarks for the levelling lines
4. HeightSetup	Computes the (B,L,H) of the setups within the levelling sections/lines
5. Accuracy_C	1. Computes the refraction coefficient ***C*** at each setup as well as the relevant average ***C***-value for the entire levelling line; 2. Performs statistical assessment of the average ***C***-values for the sections in the line.
6. Equivalent	3.1. Calculates the equivalent heights; 3.2. Computes the refraction correction based on Eq. (5) for each levelling line using the section's ***C***-values; 3.3. Calculates a correction, which includes the influence of the equivalent height; the correction is to be applied on the measured height differences per section – see Eq. (15); 3.4. Computes the levelling misclosures by sections and by levelling lines taking into account 3.2 and 3.3.
7. Difference	Computes differences between the height differences for a section obtained by forward and backward levelling and estimated their accuracy

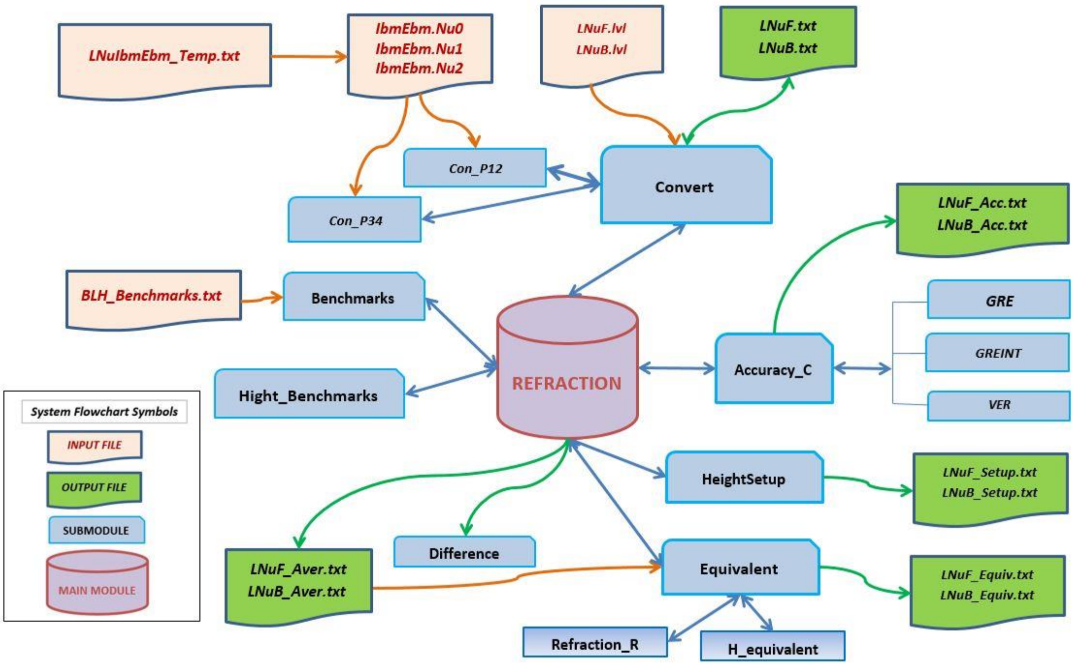


Figure 5: Flowchart of the REFRACTION package

3.3 Refraction coefficient (*C*-value) computation and accuracy estimation

3.3.1 Refraction coefficient computations

During this research, several formulae for refraction coefficient determination were discussed and tested. The best solution, which is implemented in the software package, is based on the Eq. (14). It expresses the logarithmic change of the temperature with height and corresponds to the continuous change of vertical refraction due to different terrain, ground cover, month and time of day.

Following the recommendations from the aforementioned discussions, as well as Sec. 2.3 of this paper, the calculation of the ***C***-value for each setup is considered in two cases only:

- case of normal atmosphere, where $t_1 > t_2 > t_3$ (the ***C***-values are negative), and
- case of inverse atmosphere, where $t_1 < t_2 < t_3$ (the ***C***-values are positive).

The averaged ***C***-value per section is then determined by the normal or inverse case, considering which one of the two ***C***-values is obtained using higher number of setups. When the number of negative and positive values of refraction coefficients is equal, the average coefficient for the section is calculated from the former number of values, which represents the case of normal atmosphere.

It is also considered that an appropriate level of significance for the refraction coefficient values is α = 0.3%. This corresponds to 99.7% probability for the ***C***-values to be obtained within the 3σ interval: [-1; +1].

Within the introduced restrictions, an average ***C***-value for some of the sections proved impossible to compute. In these cases, the ***C***-value in question was interpolated using the neighbouring values.

Next, average refraction coefficients referring to the middle point of each section were computed. These values were then subjected to a statistical assessment using descriptive statistics, Pearson and Kolmogorov – Smirnov Criteria.

As a next step, average ***C***-values per section from backward and forward levelling are computed. The same two cases of atmosphere (normal and inverse) were taken into account.

A sample of the results on this stage is shown in Fig. 6 and a summary for all Projects is given in Table 4. Denoting sections as 'double' or 'single' indicates whether the results refer to double runs (backward and forward levelling) or single ones due to insufficient temperature data.

Finally, ***C***-values for levelling line as a weighted mean from section ***C***-values were computed.

Some conclusions, based on the analysis of all results, are:

- Only 46% of the ***C***-values (4540 sections) are consistent with the normal atmosphere assumption; the rest of the obtained ***C***-values are positive and thus correspond to the inverse atmosphere case. Such results were expected (see Agnus-Leppan, 1984;

Webb, 1969) considering the fact that the levelling measurements and temperature observations in forward and backward direction were done during different parts of the day (morning, noon, afternoon) and, therefore, are affected by the different conditions of the atmosphere: normal, neutral and inverse.

– The obtained ***C***-values per sections are consistent in spite of being obtained through single or double runs. The range of obtained negative ***C***-values is about 0.018, and the range of the acquired positive values is about 0.017 (see Table 4).
– All weighted mean ***C***-values for levelling line have an accuracy of 0.02 regardless of their sign. The ranges of the values are (- 0.426; - 0.408) and (+ 0.422; + 0.439), respectively.

Table 4: Summary of the results for ***C***-values per section computed as average from backward and forward levelling for all levelling Projects

Type of sections	**ave *C*-values**	**STD**	**Number of sections**
Double sections with negative coefficient	**- 0.426**	0.022	1647
Single sections with negative coefficient	**- 0.408**	0.022	2893
Double sections with positive coefficient	**0.439**	0.021	1211
Single sections with positive coefficient	**0.422**	0.019	4160
Total number of sections (all Projects)			**9911**

PROJECT 1

```
---------------------------------------------------------------------------------|
                          D O U B L E   S E C T I O N S
---------------------------------------------------------------------------------|
Line  forward     backward |        |        |   forward     backward          |        |
     Nsec    C_  Nsec    C_ | C_aver| H_aver| Nsec    C+     Nsec   C+   | C+aver| H_aver|
---------------------------|-------|-------|----------------------------------|-------|
 L01   27 -0.222  27 -0.339 -0.280 | 599.22|   66   0.458   66 0.506 |   0.482| 274.32|
 L02   15 -0.328  15 -0.502 -0.415 |1325.61|   48   0.532   48 0.463 |   0.498|1157.85|
 L28   55 -0.410  55 -0.458 -0.434 |  18.96|   36   0.372   36 0.443 |   0.407|  26.26|
 L30   13 -0.284  13 -0.143 -0.214 | 531.26|    4   0.267    4 0.152 |   0.210| 164.42|
 L32   24 -0.499  24 -0.445 -0.472 |1602.79|   32   0.395   32 0.441 |   0.418|1526.20|
 L35    5 -0.230   5 -0.298 -0.264 |1005.90|   25   0.318   25 0.502 |   0.410|1027.15|
 L41  194 -0.357 194 -0.417 -0.387 |1556.30|   51   0.426   51 0.390 |   0.408| 873.52|
 L42   65 -0.266  65 -0.456 -0.361 | 599.29|   25   0.336   25 0.379 |   0.357| 645.22|
---------------------------------------------------------------------------------
      399 -0.344 399 -0.417 -0.381               302   0.423  302 0.447    0.435
---------------------------------------------------------------------------------
      Std= 0.067       0.063  0.029              Std= 0.067       0.056    0.028
---------------------------------------------------------------------------------
                          S I N G L E   S E C T I O N S
---------------------------------------------------------------------------------|
Line  forward     backward |        |        |   forward     backward          |        |
     Nsec    C_  Nsec    C_ | C_aver| H_aver| Nsec    C+     Nsec   C+   | C+aver| H_aver|
---------------------------|-------|-------|----------------------------------|-------|
 L01   15 -0.367  12 -0.401 -0.382 |1195.44|   16   0.426   76 0.447 |   0.443| 538.56|
 L02   14 -0.346  18 -0.399 -0.376 |1208.01|   19   0.475   64 0.455 |   0.459|1176.64|
 L03    4 -0.203  36 -0.367 -0.350 | 949.51|   42   0.413   24 0.403 |   0.409| 946.87|
 L28   71 -0.165  55 -0.388 -0.263 |  16.11|   59   0.379   67 0.420 |   0.401|  19.46|
 L30   35 -0.184   3 -0.399 -0.201 | 911.64|    3   0.130   39 0.313 |   0.300| 841.06|
 L32   44 -0.411  31 -0.318 -0.373 |1609.51|   32   0.490   76 0.467 |   0.474|1580.89|
 L33    2 -0.459   0  0.000 -0.459 |1284.06|    2   0.256    8 0.133 |   0.158|1254.45|
 L35   14 -0.257  10 -0.410 -0.321 |1011.54|   11   0.203   39 0.468 |   0.409|1019.68|
 L41   85 -0.320  57 -0.462 -0.377 |1258.94|   59   0.491   96 0.404 |   0.437| 900.63|
 L42   32 -0.434  86 -0.379 -0.394 | 607.87|   87   0.307   58 0.421 |   0.352| 614.61|
---------------------------------------------------------------------------------
      316 -0.294 308 -0.392 -0.342               330   0.394  547 0.422    0.412
---------------------------------------------------------------------------------
      Std= 0.099       0.040  0.034              Std= 0.084       0.053    0.031

  Total number of sections.................................. 2202
         Number of double section with negative coefficient  399
         Number of double section with positive coefficient  302
         Number of single section with negative coefficient  624
         Number of single section with positive coefficient  877
```

Figure 6: Example file with ***C***-values per section computed as average from backward and forward levelling

3.3.2 Implementation of the equivalent heights

The computation of the equivalent height h_e is performed within the REFRACTION software using Eq. (4). Its influence on the refraction is:

$$C_{equiv} = C_{ref}\left(\frac{1}{h_{e\_back}} - \frac{1}{h_{e\_for}}\right) \tag{15}$$

where C_{ref} is obtained by Eq. (5).

3.3.3 Correlation analysis of the ***C***-values

All ***C***-values per sections were tested for correlation with their spatial position and the average section equivalent height. The analysis was performed using MatLab code, which computes the correlation coefficient along with its 95% confidence interval. Summary of the results for all Projects is given in Table 5, where the significantly correlated ***C***-values are given as a count and as a percentage of the total.

Table 5: Summary of the test of significance for ***C***-values w.r.t. elevation *H* and equivalent height h_e

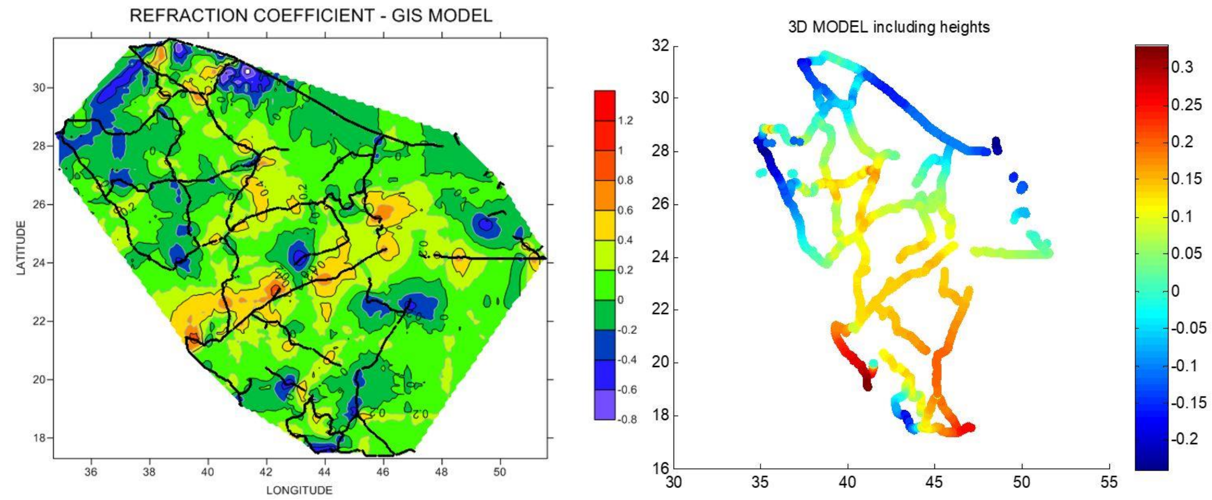
Project No	Lines	Sections	**C** (*H*)		**C** (h_e)	
			count	%	count	%
1	11	2202	10	92	4	36
2	8	1907	4	50	3	38
3	24	4162	11	46	7	29
4	14	1640	5	5	4	28
Total	**57**	**9911**	**29**	**51**	**18**	**32**

3.3.4 Results for ***C***-values for sections along forward and backward levelling lines

Results from the refraction coefficient modelling showed that the ***C***-values along the sections were fluctuating, indicating either rather dynamic changes in the air layer closest to the ground, or errors in the used temperature measuring system. In order to retrieve more signal, a moving average filter with length of ~40 km has been applied. The filtering for both forward and backward directions revealed a coherent behaviour with the raw levelling data and thus proving the presence of a real physical signal in the refraction coefficient. In addition, it was confirmed that such behaviour generally follows the Kukamiaki’s conclusion for the change of the refraction coefficient during daytime: more negative ***C***-values in the afternoon (corresponding to the normal atmospheric model) and higher chances for atmospheric inversions in the morning.

3.3.5 3D GIS models of refraction coefficient

Two 3D refraction coefficient models were computed and presented in Fig. 7.



a) 3D model based on location b) 3D model based on location and heights

Figure 7: GIS models of refraction coefficient

The first one (Fig. 7a) is location dependent, whereas the second (Fig. 7b) considers also the section's heights and in this way takes into account the second order effect of correlation between the longitude and the height and between the latitude and the height. In addition, the second model gives the best results in terms of validation (see next section) by misclosures along the levelling lines and loops. Both models can be used for precise computations and future research. The underlying shape files could be incorporated in any GIS environment and subjected to further analysis. A corresponding grid file, on the other hand, is more suitable for interpolation of observed values of refraction coefficient

3.3.6 Calculation of refraction corrections by the new formula and validation of the best solution

In order to validate the best solution for refraction coefficient model, the corresponding ***C***-values per setup were computed using the average refraction coefficient for corresponding levelling section. Computing the misclosures between forward and backward directions per levelling lines together with loop misclosures and the algebraic sum of misclosures along the loops gives an idea about the performance of the corresponding model.
After analysing misclosures along all available levelling lines, the following conclusions were made:

- The refraction corrections per setup are very sensitive to the temperature behaviour. It is recommended that average refraction coefficient and average refraction correction per section to be computed and used in case the measured temperature does not comply with its analytical logarithmic behaviour;
- The greater improvement (60% - 70% - see Fig. 8) in levelling line misclosures is obtained within the 3D refraction model dependent on height (Fig. 7b).
- The best improvement is for equivalent height refraction coefficient model reaching up to 70% per observed and 68% per Contractor's values of refraction corrections;

– The new model (Fig. 7b) for refraction corrections for both regular and equivalent height provides greater improvement comparing to Contractor’s misclosures. Therefore, this model should be recommended for precise levelling observations, expecting up to 60% decrease in levelling line misclosures.

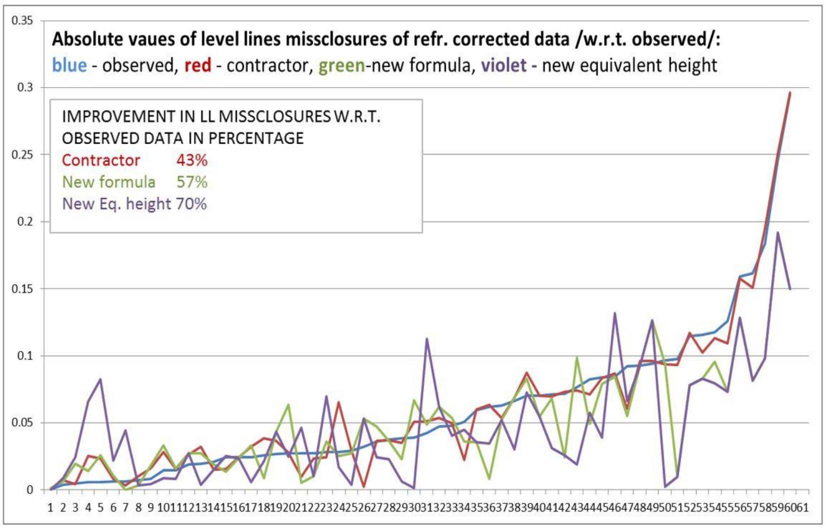


Figure 8: Example of improvement of misclosures along level lines w.r.t. observed and Contractor’s misclosures

Further investigation on the expected improvement by the new refraction model in terms of refraction coefficient effect on levelling misclosures was performed using 27 available levelling loops. Both the algebraic sums of line (forward and backward directions) and loop misclosures have been computed. Results show that the algebraic sum of misclosures (per kilometre) have an improvement of 70%. This corresponds to about 3-4 cm decrease in the misclosures’ magnitude. Similarly, the improvement in loop misclosures is 52% or 4-5 mm less than the values obtained by the Contractors. The latter could be explained by existing problems with the temperature measurements, which had led to increase of the refraction correction mean value for levelling section. Similar computations were performed taking into account the effect of the equivalent height. The results, however, show less improvement: 30% and 41%, respectively.

Thus, as a final solution the 3D GIS refraction model including height but without equivalent height refraction corrections should be considered, having greater improvement in reducing the levelling loop misclosures.

4. CONCLUSIONS AND RECOMMENDATIONS

Considering various geodetic applications and the availability of temperature measurements, four final scenarios for using refraction coefficient and/or refraction coefficient models can be recommended (Table 6). All scenarios need to be tested and validated with respect to their contribution to accuracy improvement on the entire precise levelling network in terms of misclosures along levelling lines and loops, and adjusted heights of the levelling network.

For future applications of Kukkamaki's formula, reference levels for the temperature sensors according Figure 9 should be used. In this way, the real atmospheric conditions encountered during the geodetic activities will be accounted for. The temperature measurements are needed only to determine the type of the atmosphere (normal or inverse), i.e. the sign of $\boldsymbol{C}$, which is essential for choosing among the different refraction coefficient models proposed in this paper.

The new formula for computing $\boldsymbol{C}$ – Eq. (14), could be used as well, providing that the relevant temperature measurements are obtained at reference levels of $z_1 = 0.5$ m, $z_2 = 1.5$ m and $z_3 = 2.5$ m.

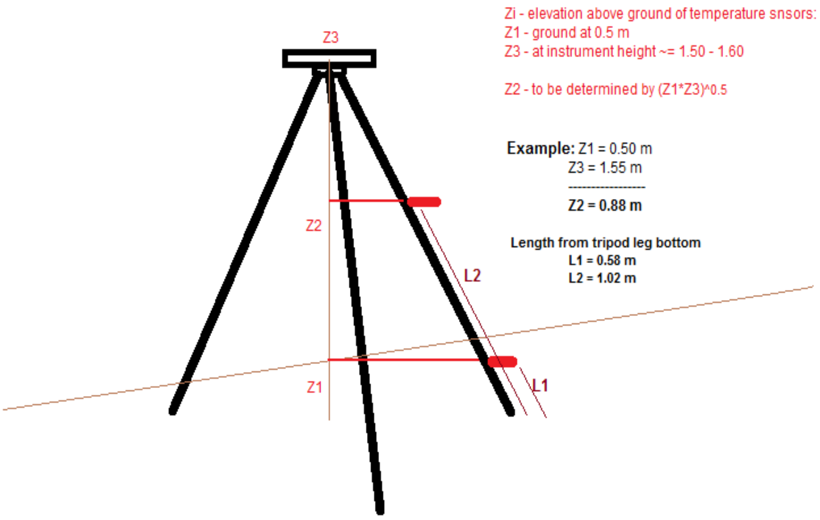


Figure 9: Proposed location of thermometers to utilize the conditions for application of Kukkamaki's formula

Table 6: Possible scenarios for application of different refraction coefficient models depending on the required accuracy and available temperature measurements

Category of geodetic applications	**Height level**	**Atmosphere**	**Refraction coefficient *C***	**Temperature measurements**
Low to mid accuracy	All heights	Not considered	**-0.408**	Not available
General high accuracy: third class precise levelling	Below 800 m	Normal ($t_3 < t_1$)	**-0.428**	at levels z_1, z_3 (see Fig. 9)
		Inverse ($t_3 > t_1$)	**+0.428**	
	Above 800 m	Normal ($t_3 < t_1$)	**-0.368**	at levels z_1, z_3 (see Fig. 9)
		Inverse ($t_3 > t_1$)	**+0.446**	
Category of geodetic applications	**Height level**	**MODEL DESCRIPTION**		**Temperature measurements**
Utilising refraction coefficient model, depending on temperature readings to determine the type	Height dependent model	$C=P_{000}+P_{100}\Delta L+P_{010}\Delta B+P_{001}H+P_{101}\Delta L.H+P_{011}\Delta B.H$, *where:* $\Delta L=L_i - 42.248^\circ$, $\Delta B=B_i - 26.260^\circ$, *with* L_i, B_i, *- navigational GPS coordinates and H of the mid-section*		at levels z_1, z_2, z_3 (see Fig. 9)

of the atmosphere (sign of ***C***) for first and second order and class precise levelling		<u>*INVERSE ATMOSPHERE*</u> *P000 = 3.3994074e-01* *P100 = 1.1976797e-03* *P010 = -3.2176302e-02* *P001 = 1.2894341e-04* *P101 = 4.5775545e-06* *P011 = 3.2209478e-05*	<u>*NORMAL ATMOSPHERE*</u> *P000 = -4.8903035e-01* *P100 = 5.4787993e-04* *P010 = -1.1974166e-02* *P001 = 4.7694739e-05* *P101 = 2.0228880e-06* *P011 = 1.1740147e-05*	
Utilising actual (due to real atmospheric conditions) ***C***-values for first and second order and class precise levelling by: **a) 3D functional model including heights** **b) 2D GIS location dependent model using interpolation facilities**	Location or height dependent models	**a) 3D - FUNCTIONAL MODEL** $C=P_{000}+P_{100}\Delta L+P_{010}\Delta B+P_{001}H \quad +P_{101}\Delta L.H+P_{011}\Delta B.H,$ *where:* $\Delta L=L_i$ *- 42.248°,* $\Delta B=B_i$ *- 26.260°, with* L_i, B_i, *- navigational GPS coordinates (B, L) and H of the mid-section.* <u>*ACTUAL ATMOSPHERE*</u> *P000 = -1.1318007e-01* *P100 = 1.7399668e-03* *P010 = -6.2398714e-02* *P001 = 2.4448398e-04* *P101 = 4.4340465e-06* *P011 = 5.9712099e-05* **b) 2D GIS LOCATION DEPENDENT MODEL USING INTERPOLATION FACILITIES**		at levels z_1, z_3 (see Fig. 9)

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