Optimum Establishment of Total Station

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SUMMARY

At least two control points must be available in order to determine the position and orientation of a total station (TS). This paper analyses optimum horizontal location of TS with respect to the control points and gives an answer to question "What is the best location of total station with respect to the control points if the goal is to determine the coordinates of TS and detail points as precisely as possible?" The optimality is deemed based on the uncertainty of the horizontal coordinates of the TS and of the points measured from it, as well as on the uncertainty of the TS orientation. The investigation of this optimality problem was performed both analytically and by the trial-and-error method.

It was found that the optimum location of TS is in the center of gravity of all control points. For a given configuration of the control and detail points, the location of TS does not influence significantly the positional uncertainty of the surveyed detail points. If the location of control points is not fixed, e.g. when using GNSS observations to establish TS, the control points should surround the working area so that the detail points to be surveyed are close to the centroid of the control points.

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1. INTRODUCTION

For most applications, the aim of total station measurements is to determine, or set out, the coordinates in a specific reference system. In order to do so, we need to establish the total station (TS), i.e. to determine its position and orientation in the given reference system. There are two methods for TS establishment: we can set up the instrument on a control point and orient it towards at least one other known point or we can set the instrument freely and measure distances and angles to at least two known points. As known points we can use existing control points or points determined with GNSS measurements. Since we can apply the same mathematical model for both methods we choose to consider the instrument set up on a known point as a special case of the free station.

It is well known that we need at least two control points to establish a TS and that we prefer to use three or more points to ensure precision and reliability of the result. But it is not so well known what is the best location of the TS in relation to the control and detail points. In this article, we will only discuss the horizontal location of the TS, because in practice, usually it is not possible to select different vertical positions for the instrument. What are the criteria for the best placement? We can judge what is "best" using the uncertainty in the station setup (uncertainty in the determined TS-coordinates and orientation) and/or using the uncertainty in the measured detail points. Controllability is also an important criterion. Good controllability means that we have a greater chance to identify and thus eliminate gross errors.

The goal of this paper is to find a location of TS that yields the least possible uncertainty and controllability in the determination of TS coordinates and orientations and in the coordinates of surveyed detail points.

2. HOW DID WE PERFORM THE ANALYSIS?

We can consider resection (free station determination) as a horizontal geodetic network. It consists of one new (TS position) and two or more control points. We measure the horizontal directions and distances from the new to the control points. The problem of finding the optimal geometry of a geodetic network is called "first order design," according to Grafarend (1974). We can solve this problem analytically or empirically. In the analytical method, we formulate optimality criterion as a function of the covariance matrix, which depends on the network geometry. By finding the minimum (or maximum) of this function, we can find the optimal geometry according to the selected criterion. The problem with the analytical method is that it is not always possible to find a closed solution, so, instead, we have to apply iterative methods for the given network.

In the empirical method, we evaluate the criterion function for a number of selected geometries and we choose the geometry, which yields minimum (or maximum) of the function. This solution is

simpler, but there is a risk is that we will not find the best solution, since it is not possible to test an infinite number of geometries. On the other hand, in practice, we need to test only the reasonable alternatives, i.e. to find the best possible solution for the near surroundings of the working area.
Here we applied both methods in order to find the optimum location of TS with respect to known and detail points.

2.1 Analytical method

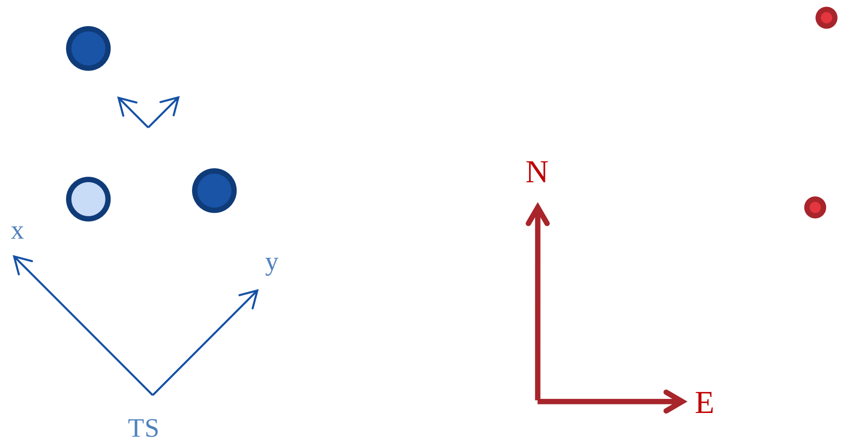
Here we describe only the basic principles; the complete mathematical model can be found in Horemuz and Jansson (2016). Our goal is to find such coordinates of TS (E_{TS}, N_{TS}), which yield the least uncertainty in the horizontal position, i.e. $u(TS) = \sqrt{\mathrm{u}^2(\mathrm{E}_{TS}) + \mathrm{u}^2(\mathrm{N}_{TS})}$ shall be minimal. The measured quantities are the horizontal distances and directions and we also assume certain level of uncertainty in the known points. It is possible to find a closed solution under assumption that all points surveyed form the TS have the same horizontal uncertainty expressed in the station's coordinate system. In other words, we assume that the uncertainty in the surveyed coordinates does not depend on the distance or direction from TS. In the following we will refer to this assumption as "equal uncertainty assumption". With this assumption, the optimum location of TS is in the centroid (mass centrum) of all known points. The analytical solution even shows that the uncertainty in the TS orientation as well as the uncertainty in the surveyed detail points does not depend on the location of TS.

The controllability (or reliability) can be computed from redundancy matrix (see e.g. Kuang 1993); its diagonal contains redundancies for each individual observation (distances, directions and coordinates of known points). Redundancy is a number between zero and one: zero redundancy means that the respective observation is not controlled by other observations; hence, it is not possible to discover eventual gross error in this observation. If a redundancy equals to one, it means that the observation is completely controlled by other observations; hence even a small gross error can be detected. In practice, all measurements towards a detail point, which is surveyed only from one instrument station, have zero redundancy. Measurements from (or towards) a point, which is surveyed from several stations, have redundancies greater than zero and less than one.

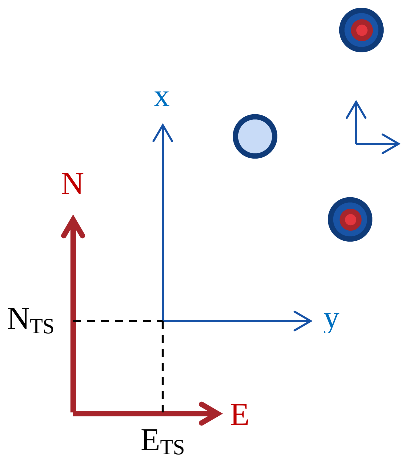
With the above mentioned "equal uncertainty assumption", the redundancies do not depend on the location of TS (E_{TS}, N_{TS}), i.e. the controllability of all individual measurements is equal for all possible E_{TS}, N_{TS}.

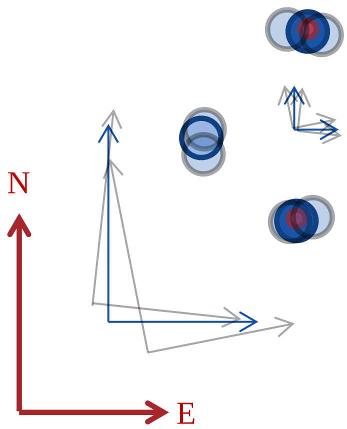
These conclusions can be visualized graphically - see Figure 1. A total station is established using two control points (red circles). TS coordinate system (blue) is defined by the instrument setup – x axis represents zero direction. The blue system is arbitrarily oriented with respect to the reference system. By measuring directions and distances towards the control points we can compute their coordinates in the blue (TS) system. These coordinates are visualized by the blue circles, which also represent the uncertainty in the horizontal position. With our "equal uncertainty assumption" all blue circles hav the same radius, no matter where is the TS located. TS establishment can be seen as transformation of the blue to the red system; the transformation parameters are estimated using the "blue" and "red" circles, which represent common points, i.e. points with known coordinates in both systems. If we establish TS several times, using independent observations, the result (TS coordinates and orientation) would vary because of the uncertainties in the measurements. Three alternative results are shown in Figure 1 d). It can be seen that the variation of the computed TS position is least in the centroid of the control points and the variation increases with the distance from the centroid. The variation in orientation is represented by the angles between the blue axes in 1 d); it is constant for all locations of TS.

Controllability or reliability can be seen as a possibility to check the fit (of the blue system to the red, according to the paragraph above). If we have only two control points, the redundancy is very small (only one redundant observation). If we make a gross error in a measurement, it would not be possible to identify the faulty measurement. The situation gets improved if we have more control points. Already with three control points we can test the fit using three combinations (point 1 and 2, 2 and 3, 1 and 3), and in such a way to identify the point with the gross error. Due to the ”equal uncertainty assumption” , the controllability will be equal for all locations of the TS.



a) b)





b) d)

Figure 1. Establishment of free station – visualization of uncertainty in horizontal position. a) Blue circles: control points surveyed in the TS coordinate system (blue axes). b) Red circles: control points in a reference system. c) Transformation of the blue system to the red one. The angle between the blue axes in a) and the red ones is TS orientation, the origin of the blue system in c) is the position of TS in the red system. d) Transformation using three different surveys of the control points (repeating TS establishment). Short blue axes: alternative location of TS.

2.2 Empirical method

What happens if the ”equal uncertainty assumption” does not hold? It reality, there is a distance-dependency in the positional uncertainty; e.g. uncertainty in the distance measurements grows with the distance. In this case there is no closed solution for the best placement of the TS; the optimization problem can be solved either by iterative analytical or by empirical method. We applied empirical method and in our numerical calculations we adopted the following standard uncertainties:

- distance u(d) = 2 mm + 2 ppm,
- horizontal direction u(ψ) = 1.5 mgon,
- control points u(CP) = 10 mm; this uncertainty in horizontal position includes even uncertainty in centering

We calculated standard uncertainties in detail point and TS coordinates, in the TS-orientation as well as the redundancies for individual measurements for different symmetrical and non-symmetrical geometries of the control points. Figure 2 shows the standard uncertainty in the TS position u(TS) and standard uncertainty in orientation u(orientation) for the simplest symmetrical case of two control points. u(TS) is the lowest in the centroid of the known points, i.e. the same results we got from the analytical method. We can also see that u(orientation) is not constant and its minimum is in the centroid. The variation in u(orientation) is very small and from the practical point of view, we can consider it as negligible.

Figure 3 shows an example of a non-symmetrical configuration of three control points. Minimum value of u(TS) is approximately 1 m, and the minimum value of u(orientation) is ca 5 m from the centroid. The difference between the lowest value and the values at the centroid is negligible, so in practice we can say that the best location of TS, which yields the least uncertainty in the station setup, is in or near the centroid.

Controllability for directions and distances is quite low; it is larger for the coordinates of the control points, which are also regarded as observations - see Figure 4. The consequence is that it is not possible to identify a gross error in an individual observation (length, direction or coordinate), but only in the point towards which the observation was made. This means that we should exclude or re-measure the point whose coordinates (E or N) has been marked as gross error. The redundancies calculated for configurations with multiple control points show the same behavior: low values for directions and distances while redundancies for the coordinates will be greater if we increase the number of known points. The variation of redundancies for different locations of the TS is negligible, meaning that the controllability is not affected by TS location.

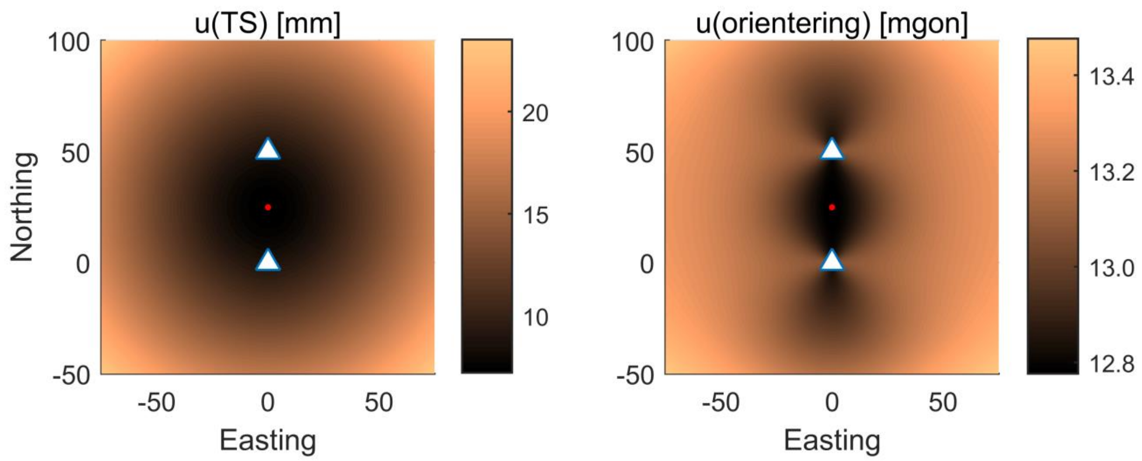


Figure 2. Standard uncertainty in position and orientation of TS. Two control points (triangles). The red dot marks the location that yields the least value.

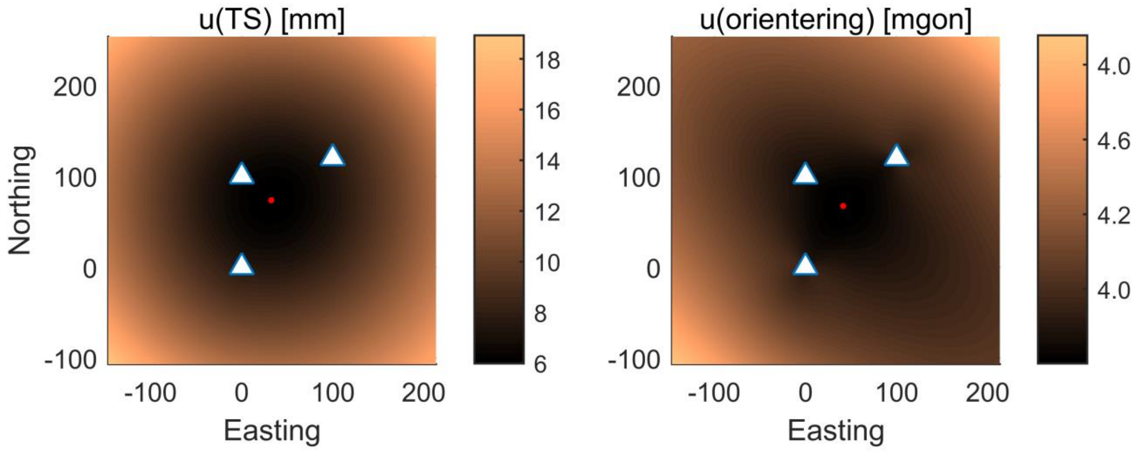


Figure 3. Standard uncertainty in position and orientation of TS. Three control points (triangles), non-symmetric geometry. The red dot marks the location that yields the least value.

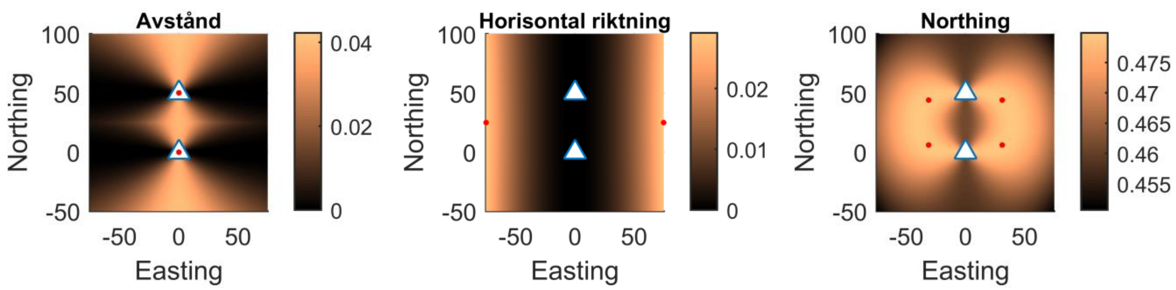


Figure 4. Redundancies for individual observations. For each TS location, the observation with least redundancy is plotted. The red dots mark the location, which yields the greatest redundancy. The redundancy for Easting coordinate is close to zero everywhere for this configuration of the control points.

We have also calculated standard uncertainty in the horizontal position of a surveyed detail point u(DP) for different configurations of control points. Figure 5 shows two configurations: one with two and one with three control points. We can see that u(DP) varies and the optimum location of TS can be at a control point, the detail point or near the centroid, depending on the number and geometry of control points and the location of detail point. Please note that the variation in u(DP) is very small, which means that the location of the TS does not affect the uncertainty in detail points. It depends only on the location of the detail point with respect to the control points. The lowest uncertainty has a detail point located in the centroid of all known points.

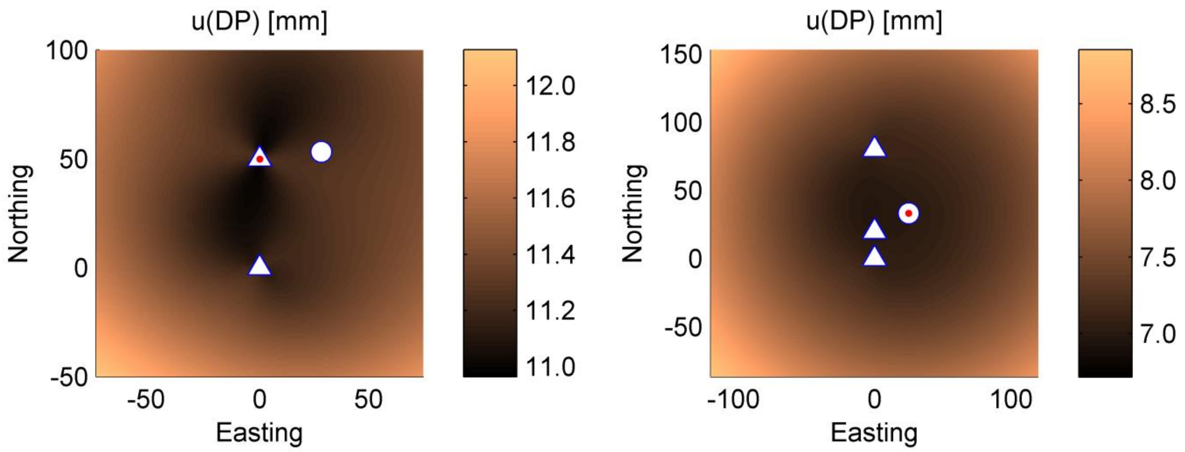


Figure 5. Standard uncertainty in horizontal position of detail point (circle). The red dot marks the location of TS that yields the least uncertainty.

3. Conclusions

Based on our analysis, we can conclude that the best location of the total station is in the centroid of the control points. This applies if we want to establish a new control point, i.e. physically mark a point under the free station, which can be used at later times. If we do not have to establish a new point, but using the free station just as a temporary setup to perform detail measurements, then it does not matter where we place the instrument (within a reasonable distance from the work area). The precision of detail measurement (with a given instrument) depends on the number of control points and their location in relation to the detail points. This is an important finding in situations where we can choose the location of known points such as when establishing TS using RTK GNSS measurements (Horemuz and Andersson 2011) or when designing control network for deformation monitoring. In such cases, we prefer to place the control points around the working area so that the detail points are near their centroid.

The reliability (controllability) of individual measurements is not affected by the instrument's location relative to the control points. Two control points provide almost no possibility to check the results regardless of where the instrument is placed. To ensure a reliable establishment, we should use at least three, preferably, more control points. It gives us a possibility to detect eventual gross errors.

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