A complete processing methodology for 3D monitoring using GNSS receivers

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Key words: Monitoring, GNSS, full variance – covariance matrix, objective weights estimation, error ellipse, error ellipsoid.

SUMMARY

GNSS measurements are widely used for the monitoring of several structures' deformations such as dams, bridges, high-rise buildings as well as landslides and earth crustal movements. In most cases the use of GNSS receivers is more convenient as it ensures continuous measurements and provides unmanned observations, long or short baselines measurement without visibility between the points. Moreover the accuracy of the static relative positioning reaches the sub-cm level.

According to the usual procedure both horizontal and vertical change vectors of each point's position are calculated in order to examine whether they should be considered as displacements or they are within the noise of the measurements.

As many commercial GNSS software packages don't provide the full variance – covariance (VCV) matrix as an output, there is often a miscalculation of the absolute and relative error ellipses or ellipsoids for any confidence level. Moreover the baselines' solution usually provides unrealistically optimistic standard errors. Thus it is often ignored or empirically scaled. The right weight estimation is needed in order to produce an objective VCV matrix from the network adjustment.

This work presents a complete, reliable processing methodology for 3d monitoring by using GNSS measurements. This processing methodology allows the use of the initial baselines measurements and leads to analytical results according to the least squares method and the law of propagation of errors.

Also the paper uses a specific technique for the preferred definition of the weights in order to be used for the unequal weight adjustment.

The network adjustment is carried out in the geocentric reference system by using linear equations and the indirect observations method. The full objective VCV matrix of the network is provided. The appropriate full rotation matrices are used in order to transform the displacement vectors as well as their variances and covariances in a local oriented plane projection in order to be more perceptible and useful.

Finally, the limitations of the proposed procedure are represented by the calculation of the difference to the error ellipses when the full VCV matrix is used.

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1. Introduction

3D monitoring using GNSS receivers is today widely used due to the convenience and the advantages that GNSS receivers provide. The 3D monitoring of dams (Lima et al, 2006), landslides (Gili et al, 2000), bridges (Barnes et al, 2003) or wide areas have some particularities that impose the use of GNSS measurements against terrestrial ones.
The use of GNSS receivers is promoted for such applications, as it is more convenient, it reduces the staff needed, it has easier instrument setting, it doesn't need visibility between the points and the measurement accuracy reaches the sub-cm level that is required for such applications.
The accurate determination of the measurements noise and the error ellipses of the control points are indispensable in order to prove displacements for a selected confidence level. Two main parameters are involved in the above calculations, the weight of each measurement and the variance –covariance (VCV) matrix which resulted from the adjustment.
For 3D monitoring, a network of control points was established in the area of interest. Measurement campaigns are carried out in selected time intervals according to the evolution of the phenomenon. The comparison of the calculated coordinates of each campaign provides the control points' displacement through time.
For each campaign (I, II…) the measurement of all the formed baselines is carried out. The relative static positioning method is used applying independent determination for each one. Only non-trivial baselines are used for the adjustment in order to ensure different conditions and to avoid bias at the calculations. Figure 1 presents a network consisting of five points, where 10 baselines are formed. By using two receivers the base lines should be measured sequentially 1-2,1-3,1-4,1-5,5-2,5-3,5-4,4-2,4-3,3-2.

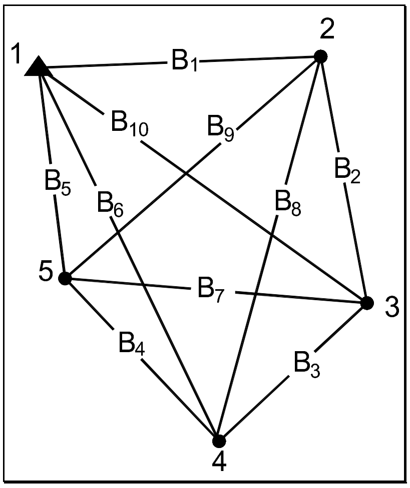


Figure 1: A typical network

The solution of each baseline provides the components $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$ in the Cartesian geocentric reference system between the occupation points i and j (Wells et al, 1986). The

standard deviations, which are essential for the network adjustment and provided by baseline solution usually are unrealistically optimistic. Thus an objective weight definition is needed.
Another significant problem emerges from the use of commercial adjustment software, as some of them do not provide the full VCV matrix as an output.
According to the proposed processing methodology firstly a reliable estimation of the weights either by three preliminary separate adjustments or by using a fast empirical estimation under some assumptions is implemented.
So, linear equations are formed and the unknown geocentric coordinates X, Y and Z are calculated by a least squares' adjustment. Thus the full VCV matrix is provided for each campaign. The differences of the coordinates $\delta X_i^{I,II}, \delta Y_i^{I,II}, \delta Z_i^{I,II}$ between successive campaigns are calculated.
A detailed rotation matrix is used for the transformation of $\delta X_i^{I,II}, \delta Y_i^{I,II}, \delta Z_i^{I,II}$ in a local projection plane $\delta E_i^{I,II}, \delta N_i^{I,II}, \delta Up_i^{I,II}$, in order to be easily comprehensible. After that the full VCV matrix $V_{\delta E,N,Up}$ is calculated according to the law of propagation of errors by using the full VCV matrix of the displacement $V_{\delta X,Y,Z}$.
Finally, by using the full VCV matrix $V_{\delta E,N,Up}$ the error ellipses or ellipsoids of the control points are calculated for the detection of the point's displacement. If covariances are not used for the error ellipse calculation, significant errors and inverse results are possible. In this case there is the possibility of a wrong decision about whether a change vector represents displacement or lies within the measurement noise.
For all the methodology's calculations the use of Excel or Matlab software is sufficient.

2. Weights estimation

During a baseline measurement, redundant measurements are collected according to the set processing interval time. Thus usually the calculated uncertainties of baseline components are overestimated, sometimes to the sub-millimetre level. Before the network adjustment, an objective estimation of the achieved uncertainties must be done, in order to form the right weight matrix.
Considering that the standard deviations of X_i, Y_i and Z_i for each point i may be different from the other and without other assumptions the following procedure may be applied.
In order to calculate the errors for each component X_i, Y_i and Z_i independently, a preliminary adjustment could be applied according to either the method of indirect observations or the method of condition equations by using the least squares method.
With the indirect observations method three independent equations systems are formed separately for X_i, Y_i and Z_i. The number of equations of each system is equal to the measured baselines. The equations have the following form:

$$\Delta X_{ij} = X_j - X_i, \quad \Delta Y_{ij} = Y_j - Y_i, \quad \Delta Z_{ij} = Z_j - Z_i \qquad (1)$$

The measured ΔX_{ij}, ΔY_{ij}, ΔZ_{ij} are used with unknowns X_i, Y_i and Z_i of the network points accordingly.
An equal weight adjustment of the above three mentioned equation systems is being performed. Three different VCV matrices V_X, V_Y, V_Z are produced independently. The

objective errors of the unknown components $e_{X_i}, e_{Y_i}, e_{Z_i}$ for each point i of the network is presented by the root of the variance for each one in V_X, V_Y, V_Z accordingly.
Also by following the linear condition equations method three independent adjustments could be produced. The condition equations are formed by network's loops closure by using the measured $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$, as follows:

$$\Delta X_{ij} + \Delta X_{jk} + \Delta X_{ki} = 0, \quad \Delta Y_{ij} + \Delta Y_{jk} + \Delta Y_{ki} = 0, \quad \Delta Z_{ij} + \Delta Z_{jk} + \Delta Z_{ki} = 0 \qquad (2)$$

It's obvious that practically the loops have always a misclosure (mc≠0)
The number of equations in every system is equal to the number of the unary loops of the network. Each loop includes three of the network's points. For the presented network in figure 1, ten unary loops are formed.
From each adjustment, a VCV matrix is produced. From this matrix, with the appropriate transformation the root of the variances for X_i, Y_i and Z_i come out. This may be considered, as previously mentioned, an objective estimation of the errors $e_{X_i}, e_{Y_i}, e_{Z_i}$ for each point i of the network.
On the other hand an empirical estimation, or the errors $e_{X_i}, e_{Y_i}, e_{Z_i}$ may be applied. For the unary loops of the network (L), the misclosure in the X, Y and Z is being calculated (mc_{loop_X} , mc_{loop_Y} , mc_{loop_Z}). Every loop's closure must be equal to zero according to the condition equation (2). The misclosure (mc) is the error, which the loop contains for three participating baselines. So a decent estimation of this error for each component is given by the following equation.

$$e_{\Delta X_{ij}} = \pm \frac{mc_{loop_X}}{\sqrt{3}} \qquad e_{\Delta Y_{ij}} = \pm \frac{mc_{loop_Y}}{\sqrt{3}} \qquad e_{\Delta Z_{ij}} = \pm \frac{mc_{loop_Z}}{\sqrt{3}} \qquad (3)$$

Then the mean errors $e_{\Delta X_m}$, $e_{\Delta Y_m}$, $e_{\Delta Z_m}$ of the baselines' components determination are calculated as follows.

$$e_{\Delta X_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta X_{ij}} \right|}{L} \qquad e_{\Delta Y_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta Y_{ij}} \right|}{L} \qquad e_{\Delta Z_m} = \pm \frac{\sum_{t=1}^{L} \left| e_{\Delta Z_{ij}} \right|}{L} \qquad (4)$$

Considering that for each baseline i, j the following equations are valid

$$e_{\Delta X_{ij}} = e_{\Delta X_m} = \pm\sqrt{e_{X_i}^2 + e_{X_j}^2} \qquad e_{\Delta Y_{ij}} = e_{\Delta Y_m} = \pm\sqrt{e_{Y_i}^2 + e_{Y_j}^2} \qquad e_{\Delta Z_{ij}} = e_{\Delta Z_m} = \pm\sqrt{e_{Z_i}^2 + e_{Z_j}^2} \qquad (5)$$

Where $e_{X_i} = e_{X_j}$, $e_{Y_i} = e_{Y_j}$, $e_{Z_i} = e_{Z_j}$

$$\text{then} \quad e_{X_i} = \pm \frac{e_{\Delta X_{ij}}}{\sqrt{2}} \qquad e_{Y_i} = \pm \frac{e_{\Delta Y_{ij}}}{\sqrt{2}} \qquad e_{Z_i} = \pm \frac{e_{\Delta Z_{ij}}}{\sqrt{2}} \qquad (6)$$

The above procedure provides an objective estimation of the errors $e_{X_i}, e_{Y_i}, e_{Z_i}$, compared to the unrealistically optimistic ones which comes out from the baselines' solution. Thus, by using these errors the weight matrix P is calculated in order to be used in the network's unequal weight adjustment.

3. The adjustment

Each baseline between the points i and j gives 3 linear equations, which have the form of equation (1). A total adjustment is applied by using all equations together and the correct weight matrix.
Thus according to the indirect observations method the following system of regular equations is formed

$$(A^T \cdot P \cdot A) \cdot x = A^T \cdot P \cdot \delta l \tag{7}$$

Where
m the number of the measured baselines
n the number of the network's points
A the matrix (3m x 3(n-1)) of the coefficients of the unknowns
δl one-columned matrix (3m x 1) of the results
x one-columned matrix (3(n-1) x 1) of the unknowns X ,Y and Z ,for each network's point
P the weight matrix (3m x 3m)
The solution of the system, which is formed in equation 7, is carried out according to equation 8. The coordinates X_i^I, Y_i^I and Z_i^I of the unknown points of the network for the I measurement campaign are calculated, considering one point as fixed.

$$\hat{x} = (A^T \cdot P \cdot A)^{-1} \cdot A^T \cdot P \cdot \delta l = N^{-1} \cdot A^T \cdot P \cdot \delta l \tag{8}$$

Also the a-posteriori rms error is given by the equation 9 as well as the variance – covariance matrix $V_{X,Y,Z}^I\ (3 \cdot (n-1) \times 3 \cdot (n-1))$ of coordinates is given by the equation10. Similar results are acquired from each measurement campaign I,II,….

$$\hat{\sigma}_o = \pm\sqrt{\frac{[U^T \cdot P \cdot U]}{n - m}} \text{ , where } U = \delta l - A \cdot \hat{x} \tag{9}$$

$$V_{X,Y,Z}^I = \hat{\sigma}_o^2 \cdot N^{-1} = \begin{bmatrix} \times & 0 & 0 & \times & 0 & 0 & .... & \times & 0 & 0 \\ 0 & \times & 0 & 0 & \times & 0 & .... & 0 & \times & 0 \\ 0 & 0 & \times & 0 & 0 & \times & .... & 0 & 0 & \times \\ \times & 0 & 0 & \times & 0 & 0 & .... & \times & 0 & 0 \\ 0 & \times & 0 & 0 & \times & 0 & .... & 0 & \times & 0 \\ 0 & 0 & \times & 0 & 0 & \times & .... & 0 & 0 & \times \\ ..... & ..... & ..... & ..... & ..... & ..... & ..... & ...... & ..... & ...... \\ \times & 0 & 0 & \times & 0 & 0 & & \times & 0 & 0 \\ 0 & \times & 0 & 0 & \times & 0 & & 0 & \times & 0 \\ 0 & 0 & \times & 0 & 0 & \times & & 0 & 0 & \times \end{bmatrix} \tag{10}$$

Where $\times$ are the positions of the non zero elements which are the variances and the co-variances of the correlated unknowns according to equation (1). The rest of the elements are zero as referred to the uncorrelated components as X_i and Y_i or X_i and Z_i or Y_i and Z_i etc under the assumption that baseline components are independent.

3.1 Absolute displacements calculation

The absolute position changes $\delta X_i^{I,II}$, $\delta Y_i^{I,II}$, $\delta Z_i^{I,II}$ of each network's point **i** between two sequential measurement campaigns (I and II) are calculated according to the following equations:

$$\delta X_i^{I,II} = X_i^{II} - X_i^{I} \qquad \delta Y_i^{I,II} = Y_i^{II} - Y_i^{I} \qquad \delta Z_i^{I,II} = Z_i^{II} - Z_i^{I} \tag{11}$$

Also a one-columned matrix $3(n-1)\times 1$, describes them.

$$\delta^{II\text{-}I} = \begin{bmatrix} \delta X_i^{I,II} \\ \delta Y_i^{I,II} \\ \delta Z_i^{I,II} \\ \delta X_{i+1}^{I,II} \\ \delta Y_{i+1}^{I,II} \\ \delta Z_{i+1}^{I,II} \\ . \\ . \\ . \\ \delta X_{n-1}^{I,II} \\ \delta Y_{n-1}^{I,II} \\ \delta Z_{n-1}^{I,II} \end{bmatrix} \tag{12}$$

The variances and covariances of $\delta X_i^{I,II}$, $\delta Y_i^{I,II}$, $\delta Z_i^{I,II}$ are presented on the VCV matrix $V_{\delta X,Y,Z}$, which is formed as the sum of the matrices $V_{X,Y,Z}^{I}$ & $V_{X,Y,Z}^{II}$ of the sequential measurement campaigns I and II, namely

$$V_{\delta X,Y,Z} = V_{X,Y,Z}^{I} + V_{X,Y,Z}^{II} \tag{13}$$

The changes $\delta X_i^{I,II}$, $\delta Y_i^{I,II}$, $\delta Z_i^{I,II}$ of each point i must be converted to an oriented local plane projection , $\delta East_i$, $\delta North_i$ and δUp_i ($\delta E_i^{I,II}$, $\delta N_i^{I,II}$, $\delta Up_i^{I,II}$) in order to be more comprehensible and to define their directions and their trends in relation to the earth's surface.

A rotation matrix S_i for each point i is calculated according to equation (14) (Bomford, 1980), (Mueller, 1969), (Soler ,1998).

$$S_i = \begin{bmatrix} -\sin\lambda_i & \cos\lambda_i & 0 \\ -\sin\varphi_i\cdot\cos\lambda_i & -\sin\varphi_i\cdot\sin\lambda_i & \cos\varphi_i \\ \cos\varphi_i\cdot\cos\lambda_i & \cos\varphi_i\cdot\sin\lambda_i & \sin\varphi_i \end{bmatrix} \tag{14}$$

Where φ_i, λ_i are the geodetic coordinates of each point of the network, which have been calculated by the GNSS receivers solution. Otherwise they could be calculated according to equation (15) (Heiskanen, Moritz, 1967)

$$\lambda_i = \arctan\left(\frac{Y_i}{X_i}\right), \quad \varphi_i = \arctan\left(\frac{Z_i + e^2\cdot N_i\cdot \sin\varphi_i}{\sqrt{X_i^2 + Y_i^2}}\right) \tag{15}$$

Where X_i, Y_i and Z_i are the Cartesian geocentric coordinates of the specific point

$$N_i = \frac{a}{\sqrt{1-e^2\sin^2\varphi_i}}, \; e^2 = 0.00669438002290, \;\; a = 6378137\text{m}$$

So, the total rotation matrix S_{ALL} for the n-1 unknown points of the network is as follows:

$$S_{ALL}=\begin{bmatrix}
-\sin\lambda_i & \cos\lambda_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\sin\varphi_i\cdot\cos\lambda_i & -\sin\varphi_i\cdot\sin\lambda_i & \cos\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\cos\varphi_i\cdot\cos\lambda_i & \cos\varphi_i\cdot\sin\lambda_i & \sin\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\lambda_{i+1} & \cos\lambda_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\varphi_{i+1}\cdot\cos\lambda_{i+1} & -\sin\varphi_{i+1}\cdot\sin\lambda_{i+1} & \cos\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \cos\varphi_{i+1}\cdot\cos\lambda_{i+1} & \cos\varphi_{i+1}\cdot\sin\lambda_{i+1} & \sin\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\lambda_{n-1} & \cos\lambda_{n-1} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\varphi_{n-1}\cdot\cos\lambda_{n-1} & -\sin\varphi_{n-1}\cdot\sin\lambda_{n-1} & \cos\varphi_{n-1}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\varphi_{n-1}\cdot\cos\lambda_{n-1} & \cos\varphi_{n-1}\cdot\sin\lambda_{n-1} & \sin\varphi_{n-1}
\end{bmatrix}$$

Thus the position changes of each point i ($\delta E_i^{I,II}, \delta N_i^{I,II}, \delta Up_i^{I,II}$) in a local projection plan are calculated according to the following equation:

$$\begin{bmatrix}\delta E_i^{I,II}\\ \delta N_i^{I,II}\\ \delta Up_i^{I,II}\\ \delta E_{i+1}^{I,II}\\ \delta N_{i+1}^{I,II}\\ \delta Up_{i+1}^{I,II}\\ .\\ .\\ .\\ \delta E_{n-1}^{I,II}\\ \delta N_{n-1}^{I,II}\\ \delta Up_{n-1}^{I,II}\end{bmatrix}=\begin{bmatrix}
-\sin\lambda_i & \cos\lambda_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\sin\varphi_i\cdot\cos\lambda_i & -\sin\varphi_i\cdot\sin\lambda_i & \cos\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\cos\varphi_i\cdot\cos\lambda_i & \cos\varphi_i\cdot\sin\lambda_i & \sin\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\lambda_{i+1} & \cos\lambda_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\varphi_{i+1}\cdot\cos\lambda_{i+1} & -\sin\varphi_{i+1}\cdot\sin\lambda_{i+1} & \cos\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \cos\varphi_{i+1}\cdot\cos\lambda_{i+1} & \cos\varphi_{i+1}\cdot\sin\lambda_{i+1} & \sin\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\lambda_{n-1} & \cos\lambda_{n-1} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\varphi_{n-1}\cdot\cos\lambda_{n-1} & -\sin\varphi_{n-1}\cdot\sin\lambda_{n-1} & \cos\varphi_{n-1}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\varphi_{n-1}\cdot\cos\lambda_{n-1} & \cos\varphi_{n-1}\cdot\sin\lambda_{n-1} & \sin\varphi_{n-1}
\end{bmatrix}\cdot\begin{bmatrix}\delta X_i^{I,II}\\ \delta Y_i^{I,II}\\ \delta Z_i^{I,II}\\ \delta X_{i+1}^{I,II}\\ \delta Y_{i+1}^{I,II}\\ \delta Z_{i+1}^{I,II}\\ .\\ .\\ .\\ \delta X_{n-1}^{I,II}\\ \delta Y_{n-1}^{I,II}\\ \delta Z_{n-1}^{I,II}\end{bmatrix}$$

The VCV matrix $V_{\delta E,N,Up}$ for the components $\delta E_i^{I,II}, \delta N_i^{I,II}, \delta Up_i^{I,II}$ are calculated according to the law of propagation of errors by using the appropriate J matrix as

$$V_{\delta E,N,Up}=J\cdot V_{\delta X,Y,Z}\cdot J^T \tag{16}$$

Where $V_{\delta X,Y,Z}$ comes from equation 13 and J matrix is formed by taking partial derivatives of the previous equation with respect to $\delta X_i^{I,II}$, $\delta Y_i^{I,II}$, $\delta Z_i^{I,II}$ as follows

$$J=\begin{bmatrix}
-\sin\lambda_i & \cos\lambda_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\sin\varphi_i\cdot\cos\lambda_i & -\sin\varphi_i\cdot\sin\lambda_i & \cos\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\cos\varphi_i\cdot\cos\lambda_i & \cos\varphi_i\cdot\sin\lambda_i & \sin\varphi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\lambda_{i+1} & \cos\lambda_{i+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\sin\varphi_{i+1}\cdot\cos\lambda_{i+1} & -\sin\varphi_{i+1}\cdot\sin\lambda_{i+1} & \cos\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \cos\varphi_{i+1}\cdot\cos\lambda_{i+1} & \cos\varphi_{i+1}\cdot\sin\lambda_{i+1} & \sin\varphi_{i+1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\lambda_{n-1} & \cos\lambda_{n-1} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\varphi_{n-1}\cdot\cos\lambda_{n-1} & -\sin\varphi_{n-1}\cdot\sin\lambda_{n-1} & \cos\varphi_{n-1}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\varphi_{n-1}\cdot\cos\lambda_{n-1} & \cos\varphi_{n-1}\cdot\sin\lambda_{n-1} & \sin\varphi_{n-1}
\end{bmatrix}$$

It is remarkable that the matrix J is identical with the matrix S_{ALL} as the coefficients of $\delta X_i^{I,II}, \delta Y_i^{I,II}, \delta Z_i^{I,II}$ remain invariant.

Continuously the change vector $\delta r_i^{I,II}$ and its bearing $b_i^{I,II}$, with respect to north, of each point i are calculated as follows (Agatza, 2005), (Hoover, 1984):

$$\delta r_i^{I,II} = \sqrt{(\delta E_i^{I,II})^2 + (\delta N_i^{I,II})^2} \tag{17}$$

$$b_i^{I,II} = \arctan\frac{\delta E_i^{I,II}}{\delta N_i^{I,II}} \tag{18}$$

For vertical displacement detection, the $\delta Up_i^{I,II}$ of each point i is compared with the $\sigma_{\delta Up_i^{I,II}}$ multiplied by z ($\sigma_{\delta Up_i^{I,II}} \cdot z$) for the selected confidence level according to the student (t) distribution. (For confidence levels 90%, 95% and 99% the values of the component z are 1.645, 1.96, 2.576 accordingly). If $\delta Up_i^{I,II} < \sigma_{\delta Up_i^{I,II}} \cdot z$ then there is no vertical displacement of the point i, otherwise point i has a vertical displacement.

The horizontal displacements could be checked by applying a general one –dimension check, without taking into consideration the vector's bearing. The change vector $\delta r_i^{I,II}$, is compared respectively to each one of $\sigma_{\delta E_i^{I,II}}, \sigma_{\delta N_i^{I,II}}$, whose outcome by the matrix $V_{\delta_{E,N,Up}}$, multiplied by the z component for the selected confidence level. If $\delta r_i^{I,II} < \sigma_{\delta E_i^{I,II}} \cdot z$ and $\delta r_i^{I,II} < \sigma_{\delta N_i^{I,II}} \cdot z$ then there is no horizontal displacement of the point i .The apparent $\delta r_i^{I,II}$ is within the noise of the measurements. Also if $\delta r_i^{I,II} > \sigma_{\delta E_i^{I,II}} \cdot \lambda$ and $\delta r_i^{I,II} > \sigma_{\delta N_i^{I,II}} \cdot \lambda$ then there is horizontal displacement of the point i. If the previous inequalities aren't valid then the full check by the ellipse drawing must be applied.

For the full check procedure the absolute error ellipse is drawn for each point i for a specific confidence level and the displacement vector of each point is over designed.

The major and the minor semi-axes of the error ellipse for the absolute position change of a point i are given by the equations (19), (20) accordingly:

$$\sigma_{u_i}^{I,II} = \sqrt{\frac{\sigma^2_{\delta E_i^{I,II}} + \sigma^2_{\delta N_i^{I,II}} + \sqrt{(\sigma^2_{\delta E_i^{I,II}} - \sigma^2_{\delta N_i^{I,II}})^2 + 4 \cdot \sigma^2_{\delta E_i^{I,II} N_i^{I,II}}}}{2}} \tag{19}$$

$$\sigma_{v_i}^{I,II} = \sqrt{\frac{\sigma^2_{\delta E_i^{I,II}} + \sigma^2_{\delta N_i^{I,II}} - \sqrt{(\sigma^2_{\delta E_i^{I,II}} - \sigma^2_{\delta N_i^{I,II}})^2 + 4 \cdot \sigma^2_{\delta E_i^{I,II} N_i^{I,II}}}}{2}} \tag{20}$$

The rotation angle θ_i of the major axis of the error ellipse, which measures clockwise from the north axis like the bearing, is given by the equation:

$$\tan 2\theta_i^{I,II} = \frac{2\sigma_{\delta E_i^{I,II} N_i^{I,II}}}{\sigma^2_{\delta N_i^{I,II}} - \sigma^2_{\delta E_i^{I,II}}} \tag{21}$$

The ellipse's axes should be multiplied by the component λ for the selected confidence level according to the chi-squared distribution (for confidence levels 90%, 95% and 99% the values of the component λ are 2.146, 2.447, 3.035 accordingly). If the displacement's vector exceeds the bounds of the ellipse then a real displacement occurred otherwise the change is within the noise of the measurements.

Finally, a total approach of the absolute displacement's check could be done by the calculation of the error ellipsoid's axes for each point (Potter, 1962). The eigenvalues λ_1, λ_2, λ_3 and the respective eigenvectors are calculated by using the VCV matrix $V_{\delta E,N,Up}$. The eigenvectors are defined as the positioning vectors of the ellipsoid axes u, v, w, namely the rotation of the ellipsoid in relation to the reference system as the eigenvalues are defined as the magnitude of the semi axes.

$$\lambda_1 = \sigma_u^2 ,\ \lambda_2 = \sigma_v^2 ,\ \lambda_3 = \sigma_w^2 \tag{22}$$

Figure 2 illustrates the ellipsoid components.

The vector $\delta R_i^{I,II}$ and its spatial directions c_i, d_i, f_i of the absolute displacement of the point i are calculated as follows (Agatza , 2005), (Hoover, 1984):

$$\delta R_i^{I,II} = \sqrt{(\delta E_i^{I,II})^2 + (\delta N_i^{I,II})^2 + (\delta Up_i^{I,II})^2} \tag{23}$$

$$\cos c_i = \frac{\delta N_i^{I,II}}{\delta R_i^{I,II}} \qquad \cos d_i = \frac{\delta E_i^{I,II}}{\delta R_i^{I,II}} \qquad \cos f_i = \frac{\delta Up_i^{I,II}}{\delta R_i^{I,II}} \tag{24}$$

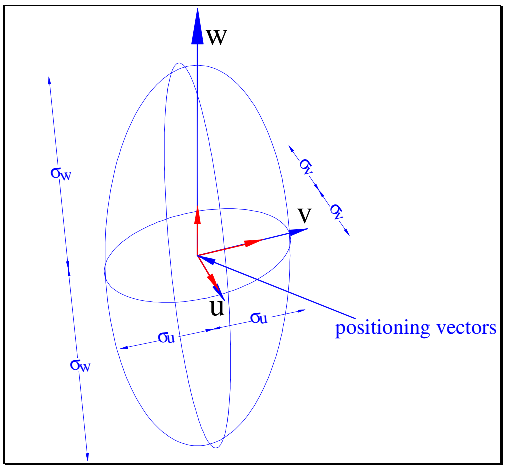


Figure 2. The error ellipsoid's components

By the comparison of the vector $\delta R_i^{I,II}$ to the error ellipsoid of each point i, it is decided if there is a displacement or the change is within the noise of the measurements.

3.2 Relative displacements calculation

In order to calculate the relative displacements between two points i and j of the network the change's vectors $\Delta\delta E_{i,j}^{I,II}$, $\Delta\delta N_{i,j}^{I,II}$, $\Delta\delta Up_{i,j}^{I,II}$ between two sequential measurement campaigns I and II are calculated by using the equations 25, 26 and 27.

$$\Delta\delta E_{i,j}^{I,II} = (E_j - E_i)^{II} - (E_j - E_i)^{I} = \delta E_j^{I,II} - \delta E_i^{I,II} \tag{25}$$

$$\Delta\delta N_{i,j}^{I,II} = (N_j - N_i)^{II} - (N_j - N_i)^{I} = \delta N_j^{I,II} - \delta N_i^{I,II} \tag{26}$$

$$\Delta\delta Up_{i,j}^{I,II} = (Up_j - Up_i)^{II} - (Up_j - Up_i)^{I} = \delta Up_j^{I,II} - \delta Up_i^{I,II} \tag{27}$$

The VCV matrix $V_{\Delta\delta E,\delta N,\delta Up}$, of the relative displacement between two points i and j is determined by the equation 28:

$$V_{\Delta\delta E,\delta N,\delta Up} = J_R \cdot V_{\delta E_{i,j},\delta N_{i,j},\delta Up_{i,j}} \cdot J_R^T \tag{28}$$

Where in this case the matrix J_R is formed by the partial derivative of the equations 25, 26, 27, with respect to $\delta E_i^{I,II}, \delta N_i^{I,II}, \delta Up_i^{I,II}\ \delta E_j^{I,II}, \delta N_j^{I,II}, \delta Up_j^{I,II}$ as follows

$$J_R = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

The VCV sub-matrix $V_{\delta E_{i,j}, N_{i,j}, Up_{i,j}}$ (3x3) is formed for both points i and j by choosing the appropriate elements from the matrix $V_{\delta E,N,Up}$ (eq. 16).

The relative change vectors $\Delta\delta r_{i,j}^{I,II}$ its bearing $b_i^{I,II}$ the major and the minor semi-axes of the relative error ellipse or ellipsoid as well as their spatial rotation for the relative position's change between two points i,j are given by the corresponding equations to 17, 18,19, 20,21,22,23 and 24 by replacing $\delta E_i^{I,II}, \delta N_i^{I,II}, \delta Up_i^{I,II}, \sigma_{\delta E_i^{I,II}}, \sigma_{\delta N_i^{I,II}}, \sigma_{\delta Up_i^{I,II}}$ with $\Delta\delta E_{i,j}^{I,II}$, $\Delta\delta N_{i,j}^{I,II}$, $\Delta\delta Up_{i,j}^{I,II}$, $\sigma_{\Delta\delta E_{ij}^{I,II}}, \sigma_{\Delta\delta N_{ij}^{I,II}}, \sigma_{\Delta\delta Up_{ij}^{I,II}}$ accordingly.

3. Discussion

The influence of the full VCV matrix is very important for the displacement determination as it makes a difference not only to the magnitude of the error ellipses' axes but also to the orientation of its main axis.

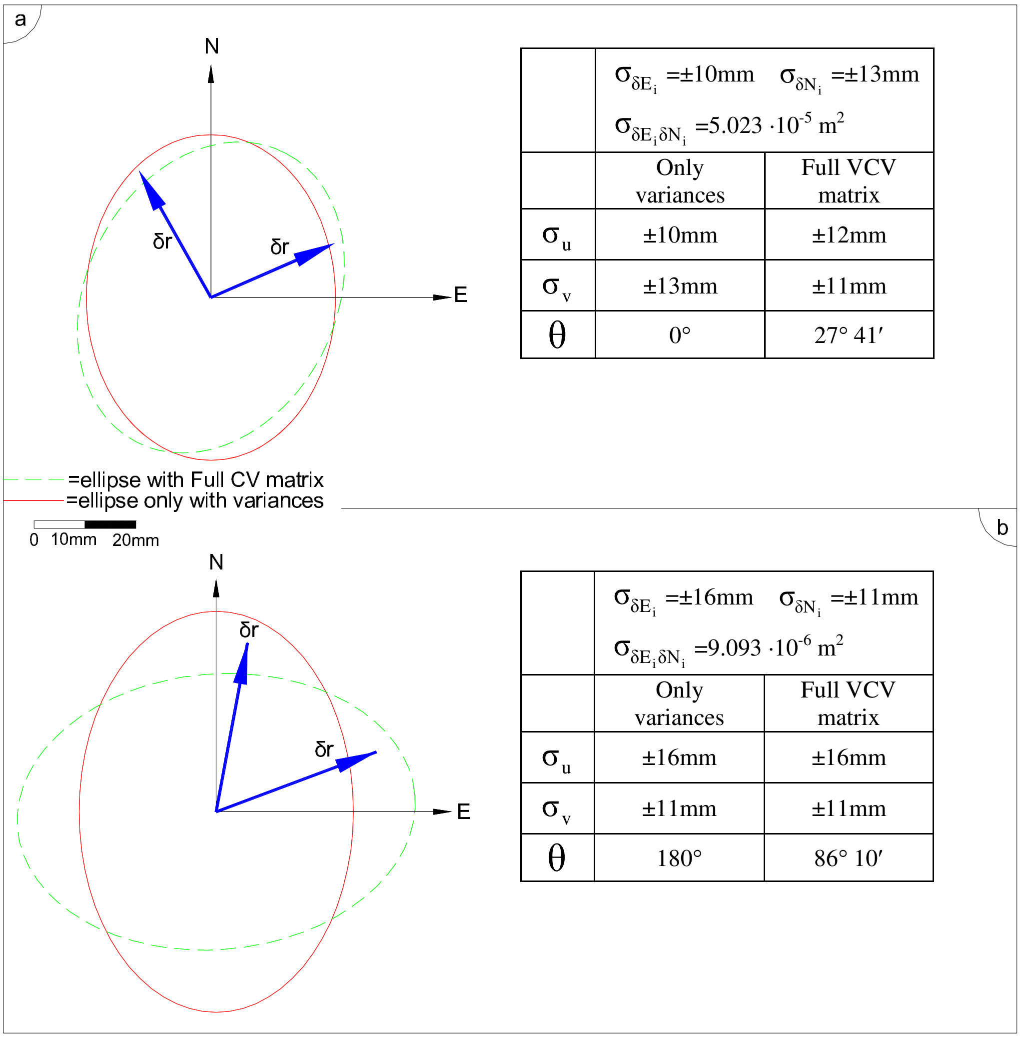
The miscalculation of the ellipse could lead to wrong conclusions about the displacements of the control point, as the displacement vector may lie accidentally outside or inside the ellipse.

Figure 3 illustrates an example of the difference in the ellipse calculation when using the full VCV matrix or not (only with variances).

If covariances aren't used, then according to equation 21 all the error ellipses have the same orientation as angle θ will be equal to zero (figure 3). That means that all the ellipses have their major axes towards north, which is totally wrong.

According to equations 19 and 20, if the covariances aren't used, the ellipse's major axis is equal to $\sigma_{\delta E_i^{I,II}}$ and the minor axis is equal to $\sigma_{\delta N_i^{I,II}}$.Thus if $\sigma_{\delta N_i^{I,II}} > \sigma_{\delta E_i^{I,II}}$ this gives an inverse result, completely different from the results when using the full VCV matrix (figure 3a).

If the size of the change vector $\delta r_i^{I,II}$ follows the inequality $\sigma_v \cdot \lambda < \delta r_i^{I,II} < \sigma_u \cdot \lambda$, for the selected confidence level, then there is a possibility to come to the wrong conclusion. According to the ellipse's axes size and the orientation, the change vector may lie inside or outside the ellipse. Thus, it could be characterised falsely as displacement or not. In figure 3 the change vector δr lies inside the red ellipse and outside the green one, thus it presents displacement when the full VCV matrix is used. On the contrary it is within the noise of the measurements if only the variances are used for the check.



	$\sigma_{\delta E_i}$ =±10mm $\sigma_{\delta N_i}$ =±13mm $\sigma_{\delta E_i \delta N_i}$ =5.023 ·10^{-5} m^2	
	Only variances	Full VCV matrix
σ_u	±10mm	±12mm
σ_v	±13mm	±11mm
θ	0°	27° 41′

	$\sigma_{\delta E_i}$ =±16mm $\sigma_{\delta N_i}$ =±11mm $\sigma_{\delta E_i \delta N_i}$ =9.093 ·10^{-6} m^2	
	Only variances	Full VCV matrix
σ_u	±16mm	±16mm
σ_v	±11mm	±11mm
θ	180°	86° 10′

Figure 3. The difference in the ellipse calculation when using the full VCV matrix for confidence level 95%.

5. Conclusions

The lack of the full VCV matrix as output, the overestimated standard errors of the baselines solution as well as the "black box" followed procedure, are the main disadvantages of the majority of commercial GNSS software when used in the 3D monitoring. The proposed processing methodology attempts to rectify this situation.

In the advantages of the method is registered the use of the initial measured data ΔX, ΔY, ΔZ in the geocentric reference system for the adjustment means no transformation uncertainties are involved. Also the linear equations, which are formed, release the procedure from approximations.

The weight definition proposed technique avoids unrealistically optimistic standard error calculation due to the GNSS ability to collect a plethora of data. Thereby, it ensures the reliability of the adjustment as it illustrates the objective achieved standard errors in the original captured data.

The use of specific rotation matrix for each point in order to calculate either the absolute or relative displacements according to the law of propagation of error ensure the correctness of the results.

The full VCV matrix formation allows the accurate error ellipse or error ellipsoid calculation, the right evaluation of the displacements and leads to safe conclusion in a way that a point's position change could be characterized as displacement or as noise of the measurements for a specific confidence level.

The comparison of the size and the rotation of the error ellipses which are formed by using the full VCV matrix or not prove that there is a strong possibility to extract different conclusions for a point's displacement. The magnitude of the ellipse axes change and its orientation is completely different. Both variations are crucial.

The one-dimension check is an overall quick check; useful in the case of small or large position changes. The full check describes better the situation as the displacement vectors are drawn in order to illustrate the magnitude and the direction of the movement. The total check by the ellipsoid drawing, gives the total replay of the spatial point's movement.

Finally, the entire procedure can be carried out in an easy Excel or Matlab environment as simple linear equations systems are solved thus no special software development is required.

The proposed processing methodology has many advantages compared to commercial softwares such as the total surveillance of the adjustment's steps, the objective weights definition and the full VCV matrix formation. Thus it is evaluated as efficient and reliable for such a trustable and serious activity as 3D monitoring by using GNSS receivers.

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  - Surveying and documenting monuments
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