PREDICTION OF DAM DEFORMATION USING KALMAN FILTER TECHNIQUE

By
DR. R. EHIGIATOR IRUGHE,
DR. Jacob Odeh EHIOROBO
AND
DR. (MRS) M.O. EHIGIATOR

INTRODUCTION

- In this study Kalman filtering technique was used in predicting current estimates of Dam deformation using two previous GPS measurements carried out in 2007 and 2008 respectively. The Kalman filter equation was then used to compute the velocity and acceleration of the Dam object.

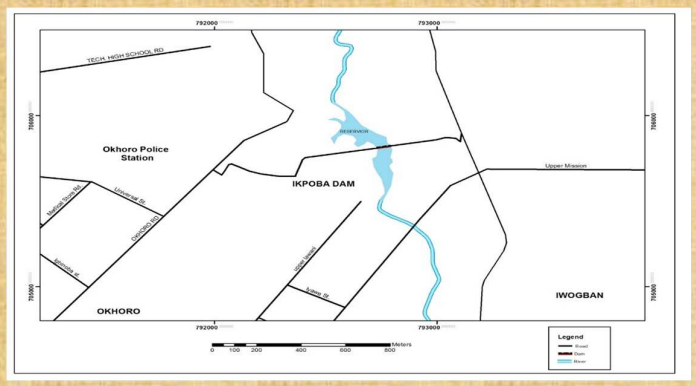
DESCRIPTION OF STUDY AREA

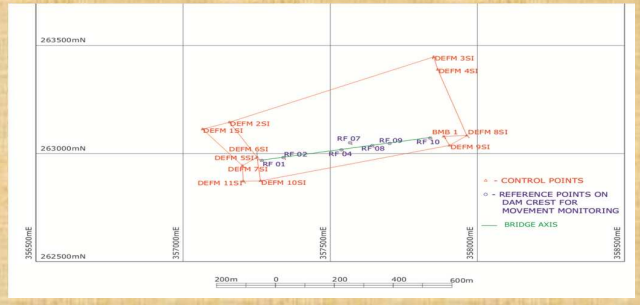
- The Ikpoba River Dam is located in Benin City, the capital of Edo State of Nigeria. The Dam together with its head works is located about 6km from the city centre (see fig 1)
- The Ikpoba Dam water supply scheme was designed to supply 160,000,000 litres of water per day at ultimate capacity. This account for about 60% of the water supply requirement for Benin City with a population of about 1.5 million.

Dam location and Reference control net work

The network for Deformation monitoring consist of eleven control points located at the upstream and downstream. Nine monitoring points was also established on the Dam

- **Location & Controls**





KINEMATIC DEFORMATION MODEL: THE KALMAN FILTER

- Kinematic deformation model determines displacements, velocity and acceleration and is time dependent. A time dependent 3-D Kinematic model that contain position, velocity and acceleration can be expressed using the following equations:

KINEMATIC DEFORMATION MODEL: THE KALMAN FILTER

$$X_j^{(k+1)} = X_j^{(k)} + (t_{k+1} - t_k)\, v_{xj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{xj}$$

$$Y_j^{(k+1)} = Y_j^{(k)} + (t_{k+1} - t_k)\, v_{yj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{yj} \qquad (1.0)$$

$$Z_j^{(k+1)} = Z_j^{(k)} + (t_{k+1} - t_k)\, v_{zj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{zj}$$

Where $X_J^{K+1}, Y_J^{K+1}, Z_J^{K+1}$ - Coordinates of point J at time t_{k+1} (predicted values), $v_{X_J}^K, v_{Y_J}^K, v_{Z_J}^K$ – velocities of X, Y,Z coordinates of point J at time t_k; $a_{X_J}^K, a_{Y_J}^K, a_{Z_J}^K$ – accelerations of X, Y,Z coordinates of point J at time t_k. k=1, 2, . . . , m (m: measurement period number(number of epochs)). j =1, 2, n (n: number of points).

Kalman Filter

- In the dynamic model of the filtering, three states with nine variables and three linear degrees of freedom (position vector), the corresponding velocity variables (velocity Vector) and the acceleration variables (acceleration vector) are considered. The state velocity and acceleration models can be written as:

Velocity and Acceleration

$$\left.\begin{aligned} v_{X_J}^{K+1} &= \frac{X_J^{K+1} - X_J^K}{\Delta t_{k+1,k}}; \\ v_{Y_J}^{K+1} &= \frac{Y_J^{K+1} - Y_J^K}{\Delta t_{k+1,k}}; \\ v_{Z_J}^{K+1} &= \frac{Z_J^{K+1} - Z_J^K}{\Delta t_{k+1,k}}. \end{aligned}\right\}$$

$$\left.\begin{aligned} a_{X_J}^{K+1} &= \frac{X_J^{K+1} - X_J^K}{\Delta t_{k+1,k}^2}; \\ a_{Y_J}^{K+1} &= \frac{Y_J^{K+1} - Y_J^K}{\Delta t_{k+1,k}^2}; \\ a_{Z_J}^{K+1} &= \frac{Z_J^{K+1} - Z_J^K}{\Delta t_{k+1,k}^2}. \end{aligned}\right\}$$

$$\left.\begin{aligned} v_{xyz_J}^{k+1} &= \frac{\sqrt{(X_J^{K+!} - X_J^K)^2 + (Y_J^{K+1} - Y_J^K)^2}}{\Delta t_{(K+1),K}} \\ d_{xyz_J}^{k+1} &= \frac{\sqrt{(Z_J^{K+!} - Z_J^K)^2}}{\Delta t_{(K+1),K}} \end{aligned}\right\}$$

$$\left.\begin{aligned} d_{xyz_J}^{k+1} &= \frac{\sqrt{(X_J^{K+!} - X_J^K)^2 + (Y_J^{K+1} - Y_J^K)^2}}{\Delta t^2_{(K+1),K}} \\ d_{xyz_J}^{k+1} &= \frac{\sqrt{(Z_J^{K+!} - Z_J^K)^2}}{\Delta t^2_{(K+1),K}} \end{aligned}\right\}$$

NUMERICAL APPLICATION OF THE KALMAN FILTER

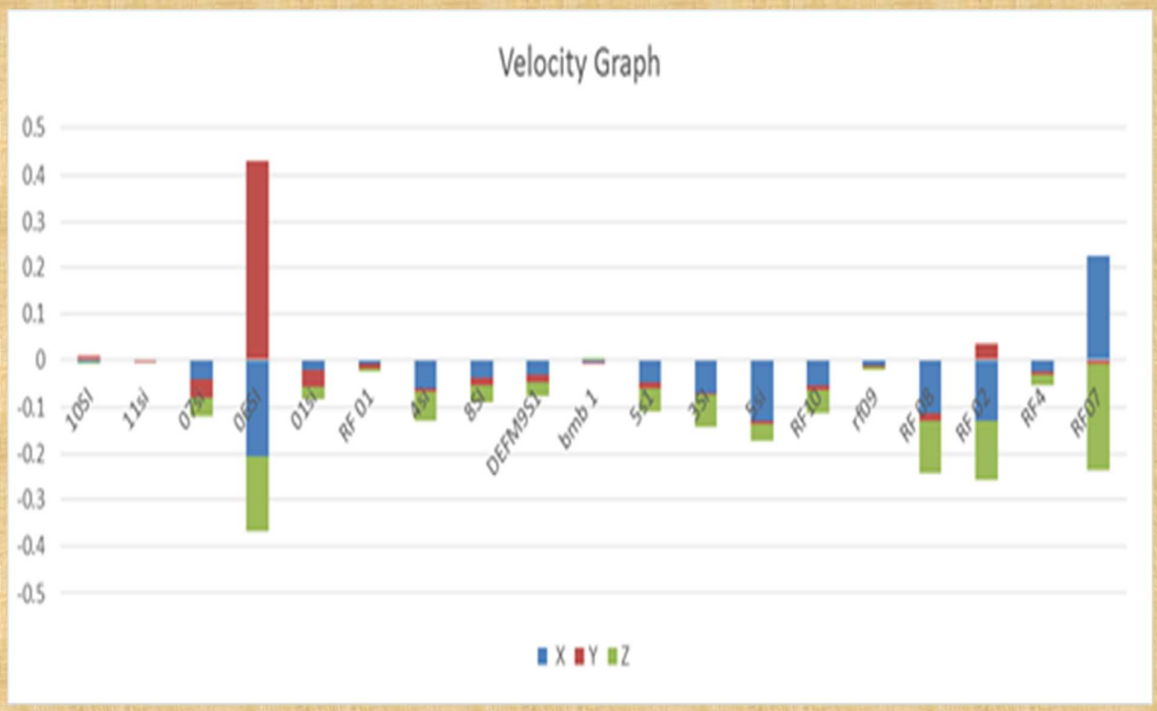
- In the first instance, static deformation analysis was carried out by evaluating the post adjustment coordinates together with the variance – covariance matrix. Next Kinematic deformation analysis based on Kalman filter technique was implemented on a MATALB software.
- The solution obtained from the Kinematic model using Kalman filter were compared with those obtained from the initial static deformation measurement results. Finally, the velocity and acceleration of the movement for each point in the network was plotted.

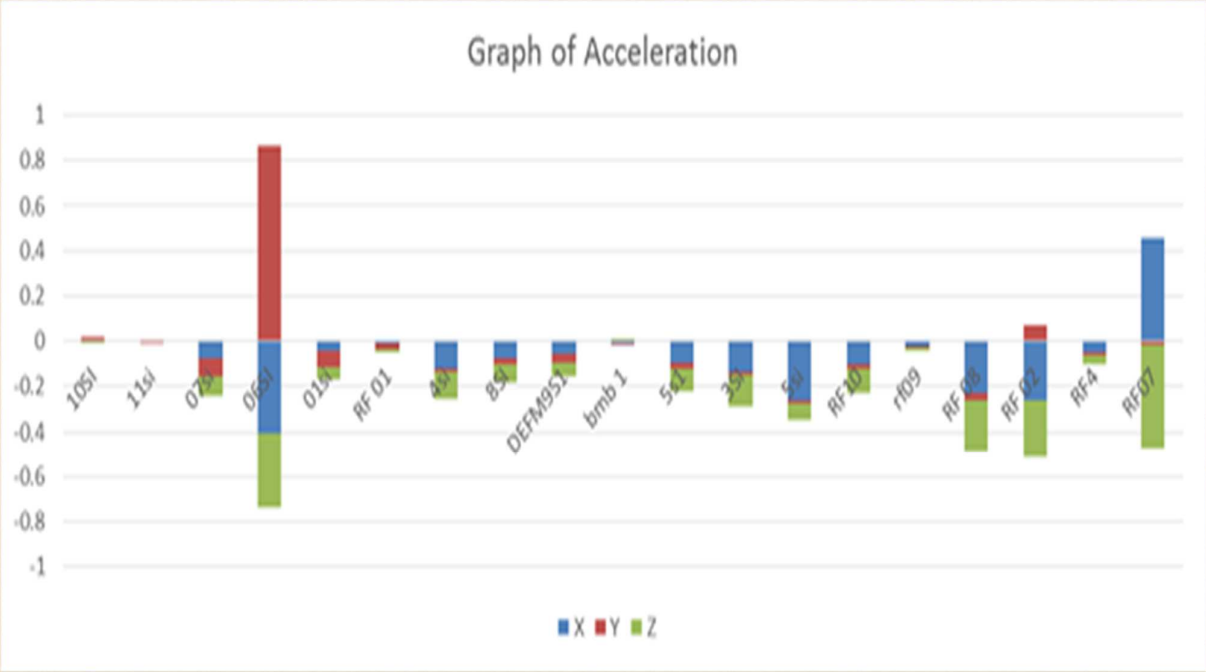
RESULTS

- The computed coordinates from the static GPS measurement results along with the velocity and acceleration of motion for each of the points is shown below.
- The velocity and acceleration of motion for each points in the network in X, Y, Z directions, the predicted coordinates using the Kinematic model for each of the points for are presented. A comparison of the measured and predicted coordinates for 2008 are also presented.

	2007 Measurement			Displacement			2008 Measurement			Velocity			Acceleration		
Neme	North1	East1	Elev1.	ΔN	ΔE	ΔZ	North2	East2	Elev2.	X	Y	Z	X	Y	Z
10SI	262870.5510	357263.0960	39.8850	-0.0011	0.0051	-0.0006	262870.5499	357263.1011	39.8844	-0.0022	0.0102	-0.0012	-0.0044	0.0204	-0.0024
11si	262868.8650	357204.4860	44.2180	0.0001	-0.0001	0.0000	262868.8651	357204.4859	44.2180	0.0002	-0.0002	0.0000	0.0004	-0.0004	0.0000
07si	262941.0620	357201.6470	44.0220	-0.0193	-0.0199	-0.0210	262941.0427	357201.6271	44.0010	-0.0386	-0.0398	-0.0420	-0.0772	-0.0796	-0.0840
06SI	262979.7740	357251.3880	39.5500	-0.1024	0.2157	-0.0822	262979.6716	357251.6037	39.4678	-0.2048	0.4314	-0.1644	-0.4096	0.8628	-0.3288
01si	263110.1860	357066.4360	50.3060	-0.0109	-0.0181	-0.0124	263110.1781	357066.4179	50.2936	-0.0218	-0.0362	-0.0248	-0.0436	-0.0724	-0.0496
RF 01	262965.1170	357267.5310	38.4610	-0.0027	-0.0065	-0.0032	262965.1143	357267.5247	38.4578	-0.0054	-0.0126	-0.0064	-0.0108	-0.0252	-0.0128
4si	263386.1340	357865.5420	39.4500	-0.0298	-0.0038	-0.0804	263386.1042	357865.5382	39.4196	-0.0596	-0.0076	-0.0608	-0.1192	-0.0152	-0.1216
8SI	263080.3670	357964.0030	42.8980	-0.0182	-0.0080	-0.0184	263080.3488	357963.9950	42.8796	-0.0364	-0.0160	-0.0368	-0.0728	-0.0320	-0.0736
DEFM9S1	263085.9700	357904.9820	39.2710	-0.0143	-0.0098	-0.0143	263085.9557	357904.9722	39.2567	-0.0286	-0.0196	-0.0286	-0.0572	-0.0392	-0.0572
bmb 1	263076.9420	357885.7800	38.3150	-0.0016	-0.0010	0.0023	263076.9404	357885.7790	38.3173	-0.0032	-0.0020	0.0046	-0.0064	-0.0040	0.0092
5s1	263175.6980	357933.4640	40.1760	-0.0231	-0.0076	-0.0239	263175.6749	357933.4564	40.1521	-0.0462	-0.0152	-0.0478	-0.0924	-0.0304	-0.0956
3SI	263444.5660	357851.6970	40.0370	-0.0344	-0.0026	-0.0348	263444.5316	357851.6944	40.0022	-0.0688	-0.0052	-0.0696	-0.1376	-0.0104	-0.1392
5sI	263175.7010	357933.4640	40.2110	-0.0645	-0.0040	-0.0184	263175.6365	357933.4600	40.1926	-0.1290	-0.0080	-0.0368	-0.2580	-0.0160	-0.0736
RF10	263068.5370	357839.7480	37.9670	-0.0260	-0.0051	-0.0253	263068.5110	357839.7429	37.9417	-0.0520	-0.0102	-0.0506	-0.1040	-0.0204	-0.1012
rf09	263050.9680	357741.3140	37.8450	-0.0050	-0.0022	-0.0036	263050.9630	357741.3118	37.8414	-0.0100	-0.0044	-0.0072	-0.0200	-0.0088	-0.0144
RF 08	263083.9000	357642.8350	37.8190	-0.0566	-0.0081	-0.0571	263083.8434	357642.8269	37.7619	-0.1132	-0.0162	-0.1142	-0.2264	-0.0324	-0.2284
RF 02	262978.5480	357341.2900	37.9150	-0.0645	0.0173	-0.0641	262978.4785	357341.3073	37.8509	-0.1290	0.0346	-0.1282	-0.2580	0.0692	-0.2564
RF4	263014.3530	357537.9720	37.9250	-0.0125	-0.0024	-0.0113	263014.3405	357537.9696	37.9137	-0.0250	-0.0048	-0.0226	-0.0500	-0.0096	-0.0452
RF07	263047.2180	357567.5600	37.9020	0.1130	-0.0040	-0.1135	263047.3310	357567.5560	37.7885	0.2260	-0.0080	-0.2270	0.4520	-0.0160	-0.4540

GPS measurement Results for 2007 and 2008 measurement period with velocity and acceleration.





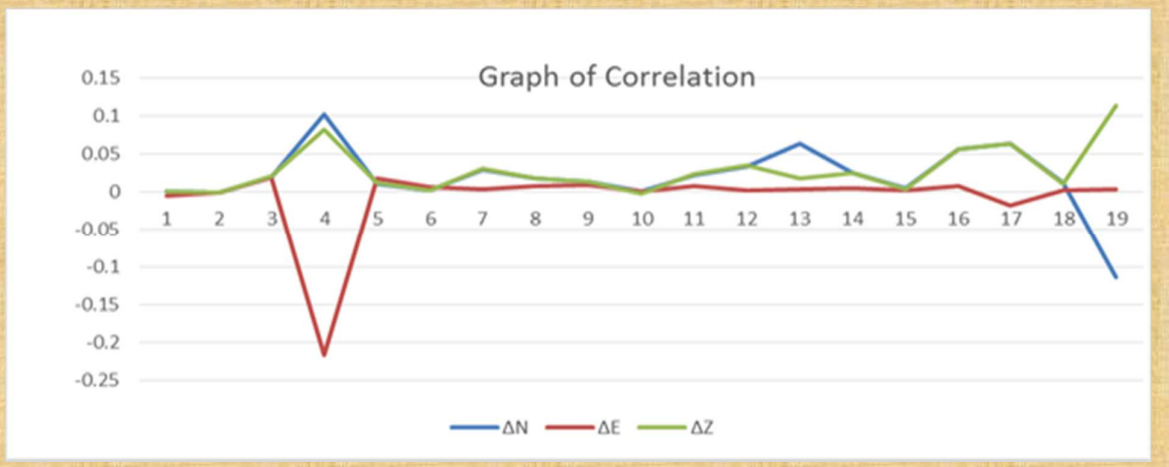
Prediction

	Prediction for 2008			Prediction for 2009			Prediction for 2010			Prediction for 2011			Prediction for 2012			Prediction for 2013			
Name	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
10SI	262870.5488	357263.1062	39.8838	262870.5444	357263.1266	39.8814	262870.5378	357263.1572	39.8778	262870.5290	357263.1980	39.8730	262870.5180	357263.2490	39.8670	262870.5048	357263.3102	39.8598
11si	262868.8652	357204.4858	44.2180	262868.8656	357204.4854	44.2180	262868.8662	357204.4848	44.2180	262868.8670	357204.4840	44.2180	262868.8680	357204.4830	44.2180	262868.8692	357204.4818	44.2180
07si	262941.0234	357201.6072	43.9800	262940.9462	357201.5276	43.8960	262940.8304	357201.4082	43.7700	262940.6760	357201.2490	43.6020	262940.4830	357201.0500	43.3920	262940.2514	357200.8112	43.1400
06SI	262979.5692	357251.8194	39.3856	262979.1596	357252.6822	39.0568	262978.5452	357253.9764	38.5636	262977.7260	357255.7020	37.9060	262976.7020	357257.8590	37.0840	262975.4732	357260.4474	36.0976
01si	263110.1672	357066.3998	50.2812	263110.1236	357066.3274	50.2316	263110.0582	357066.2188	50.1572	263109.9710	357066.0740	50.0580	263109.8620	357065.8930	49.9340	263109.7312	357065.6758	49.7852
RF 01	262965.1116	357267.5184	38.4546	262965.1008	357267.4932	38.4418	262965.0846	357267.4554	38.4226	262965.0630	357267.4050	38.3970	262965.0360	357267.3420	38.3650	262965.0036	357267.2664	38.3266
4si	263386.0744	357865.5344	39.3892	263385.9552	357865.5192	39.2676	263385.7764	357865.4964	39.0852	263385.5380	357865.4660	38.8420	263385.2400	357865.4280	38.5380	263384.8824	357865.3824	38.1732
8SI	263080.3306	357963.9870	42.8612	263080.2578	357963.9550	42.7876	263080.1486	357963.9070	42.6772	263080.0030	357963.8430	42.5300	263079.8210	357963.7630	42.3460	263079.6026	357963.6670	42.1252
DEFM9S1	263035.9414	357904.9124	39.2424	263035.8842	357904.8732	39.1852	263035.7984	357904.8144	39.0994	263035.6840	357904.7360	38.9850	263035.5410	357904.6380	38.8420	263035.3694	357904.5204	38.6704
bmb 1	263076.9388	357885.7780	38.3196	263076.9324	357885.7740	38.3288	263076.9228	357885.7680	38.3426	263076.9100	357885.7600	38.3610	263076.8940	357885.7500	38.3840	263076.8748	357885.7380	38.4116
5s1	263175.6518	357933.4488	40.1282	263175.5594	357933.4184	40.0326	263175.4208	357933.3728	39.8892	263175.2360	357933.3120	39.6980	263175.0050	357933.2360	39.4590	263174.7278	357933.1448	39.1722
3SI	263444.4972	357851.6918	39.9674	263444.3596	357851.6814	39.8282	263444.1532	357851.6658	39.6194	263443.8780	357851.6450	39.3410	263443.5340	357851.6190	38.9930	263443.1212	357851.5878	38.5754
5si	263175.5720	357933.4560	40.1742	263175.3140	357933.4400	40.1006	263174.9270	357933.4160	39.9902	263174.4110	357933.3840	39.8430	263173.7660	357933.3440	39.6590	263172.9920	357933.2960	39.4382
RF10	263068.4850	357839.7378	37.9164	263068.3810	357839.7174	37.8152	263068.2250	357839.6868	37.6634	263068.0170	357839.6460	37.4610	263067.7570	357839.5950	37.2080	263067.4450	357839.5338	36.9044
rf09	263050.9480	357741.3096	37.8378	263050.9280	357741.3008	37.8234	263050.8980	357741.2876	37.8018	263050.8580	357741.2700	37.7730	263050.8080	357741.2480	37.7370	263050.7480	357741.2216	37.6938
RF 08	263033.3868	357642.8188	37.7048	263033.1604	357642.7864	37.4764	263032.8208	357642.7378	37.1338	263032.3680	357642.6730	36.6770	263031.8020	357642.5920	36.1060	263031.1228	357642.4948	35.4208
RF 02	262978.4140	357341.3246	37.7868	262978.1560	357341.3938	37.5304	262977.7690	357341.4976	37.1458	262977.2530	357341.6360	36.6330	262976.6080	357341.8090	35.9920	262975.8340	357342.0166	35.2228
RF4	263014.3280	357537.9672	37.9024	263014.2780	357537.9576	37.8572	263014.2030	357537.9432	37.7894	263014.1030	357537.9240	37.6990	263013.9780	357537.9000	37.5860	263013.8280	357537.8712	37.4504
RF07	263047.4440	357567.5520	37.6750	263047.8960	357567.5360	37.2210	263048.5740	357567.5120	36.5400	263049.4780	357567.4800	35.6320	263050.6080	357567.4400	34.4970	263051.9640	357567.3920	33.1350

Prediction and Correlation

	2008 Measurement			Prediction for 2008			Correlation		
Name	North2	East2	Elev2.	X	Y	Z	ΔN	ΔE	ΔZ
10SI	262870.5499	357263.101	39.8844	262871	357263	39.8838	0.0011	-0.0051	0.0006
11si	262868.8651	357204.486	44.218	262869	357204	44.218	-0.0001	0.0001	0
07si	262941.0427	357201.627	44.001	262941	357202	43.98	0.0193	0.0199	0.021
06SI	262979.6716	357251.604	39.4678	262980	357252	39.3856	0.1024	-0.2157	0.0822
01si	263110.1781	357066.418	50.2936	263110	357066	50.2812	0.0109	0.0181	0.0124
RF 01	262965.1143	357267.525	38.4578	262965	357268	38.4546	0.0027	0.0063	0.0032
4si	263386.1042	357865.538	39.4196	263386	357866	39.3892	0.0298	0.0038	0.0304
8SI	263080.3488	357963.995	42.8796	263080	357964	42.8612	0.0182	0.008	0.0184
DEFM9S1	263035.9557	357904.922	39.2567	263036	357905	39.2424	0.0143	0.0098	0.0143
bmb 1	263076.9404	357885.779	38.3173	263077	357886	38.3196	0.0016	0.001	-0.0023
5s1	263175.6749	357933.456	40.1521	263176	357933	40.1282	0.0231	0.0076	0.0239
3SI	263444.5316	357851.694	40.0022	263444	357852	39.9674	0.0344	0.0026	0.0348
5si	263175.6365	357933.46	40.1926	263176	357933	40.1742	0.0645	0.004	0.0184
RF10	263068.511	357839.743	37.9417	263068	357840	37.9164	0.026	0.0051	0.0253
rf09	263050.953	357741.312	37.8414	263051	357741	37.8378	0.005	0.0022	0.0036
RF 08	263033.4434	357642.827	37.7619	263033	357643	37.7048	0.0566	0.0081	0.0571
RF 02	262978.4785	357341.307	37.8509	262978	357341	37.7868	0.0645	-0.0173	0.0641
RF4	263014.3405	357537.97	37.9137	263014	357538	37.9024	0.0125	0.0024	0.0113
RF07	263047.331	357567.556	37.7885	263047	357568	37.675	-0.113	0.004	0.1135

Graph of correlation for year 2008



DISCUSSIONS

- Analysis of the results between the measurement update and predicted deformation results for 2008 indicate a correlation between the two, except in the case of 6SI and RF 7 where the correlation was weak.

CONCLUSION

- In this study, the Kalman filter technique based on Kinematic Deformation analysis was applied to measurement data collection by static GNSS at the Ikpoba River Dam in Benin City, Nigeria.
- By comparing the predicted and measured displacements, the efficiency of the Kinematic deformation model using Kalman filter was demonstrated. A major advantage in the method is the ability to carry out step wise computation of structural movement parameters which projects forward the expected deformation at any later time.
- The graph of correlation reveals the accuracy of the predicted deformation values which compared quite well with the measured deformation for 2008. Further research is on going in order to determine the behavior for longer prediction period based on measured displacement.

THANKS FOR YOUR ATTENTION