GIS Based Prescriptive Model for Solving Optimal Land Allocation

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Key words: Prescriptive model, Optimization, GIS, Site suitability, Linear programming.

SUMMARY

GIS prescriptive modelling incorporates mathematical optimization techniques with geographic information systems. This paper described an integration of GIS based suitability model with zero-one linear integer programming for prescriptive modelling of land use allocation. The objective is to identify optimal regions for new residential land that minimized total development cost. The decision variables are formulated using feasible regions that were derived from GIS based suitability model. The constraints are spatial attributes representing the area, number of regions, land suitability value, proximity, and heights. The optimal solution will be combination of zeros and ones of the decision variables for which the objective is optimized whilst maintaining feasibility in terms of the constraints. A series of test was performed using mixed integer branch and bound algorithm to evaluate the optimal feasible sites for the residential land. The sensitivity test conducted on the model properties through changes in the input variables indicates consistency on the model outputs. The model can be easily integrated with GIS based suitability model in finding the optimal solution for other specific land allocation problem.

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1. INTRODUCTION

GIS based suitability model determines the suitability of a given location for a defined use on the basis of its physical, economic, social, and environmental characteristics. This model has turn into a standard part of planning analysis at many scales. It was developed in order to connect spatially independent factors within the environment and, consequently, to provide a more unitary view of their interactions (Steiner et al. 2000). The model has been widely used in a variety of planning and decision-making situations including landfill sitting (Natesan and Suresh 2002, Bilgehan et al., 2010, Giancarlo et al., 2009), waste disposal management (Basnet et al., 2001), recreational facilities setting (Kliskey, 2000), coastal zone management (Fabbri, 1998), and definition of protected areas (Villa et al., 2002). A comprehensive and critical monograph surveys techniques for GIS-based suitability mapping and modelling has been described by Malczewski (2004). Such techniques include the Boolean overlay operations for conjunctive and disjunctive screening of feasible alternatives (Malczewski, 2000), weighted linear combinations and simple additive weighting (Eastman, et al.1995), ideal point methods (Anchen, et al. 1997), concordance analysis (Joerin et al. 2001), analytical hierarchy processes (AHPs) (Forman and Gass, 2001), and ordered weighted averaging (Jiang and Eastman 2000).

A main drawback of this model is that it cannot determine the most optimal site amongst feasible locations. Optimal site selection is a problem of selecting, from a set of feasible sites, the group of sites that best achieves a pre-specified set of goals or objectives, within specific constraints (Robert et al., 2000). In engineering terms, it is the process of selecting a set of sites among a set of spatial locations. This problem is normally accounted in infrastructure planning sectors, for example, in the planning of new towns, the location of a new facility, or the precise selection of a construction site. To encounter this problem, a prescriptive form of defining the most optimal location has to be addressed. Prescriptive modeling in land use planning attempt to "prescribe or optimize land use patterns to meet desired planning goals subject to various physical, environmental, economical, and social constraints" (Sharpe et al., 1982,). In other word, prescriptive models typically have objective function(s) that provides the criterion for optimizing a system and generally developed using various mathematical programming techniques namely linear or non-linear programming, and integer programming (Riveira and Maseda, 2006).

2. GIS PRESCRIPTIVE MODEL

GIS prescriptive models are developed through integrating mathematical optimization techniques for the acquisition of attribute data with geographic information systems for mapping of the results. Chuvieco (1993) introduced linear programming as a tool for spatial modelling within a GIS to solve the minimisation of rural unemployment using technical, financial and ecological constraints. GIS models in the optimization of land use allocation problems involve proficient distribution of activities over feasible sites in order to meet demand and maintain physical, economic, environmental, or social constraints. Models involving allocation of spatial activities are not distinctive. It extent over areas such as urban and regional planning, forest management, reserve design, site restoration, facility location, land acquisition, or waste landfill siting (Aerts et al., 2003, Williams 2002, Williams and ReVelle 1996, Benabdallah and Wright 1992, Gilbert et al., 1985, Tomlin and Johnston 1988, Wright et al. 1983). Eastman et al., (1995) developed a decision support module for solving land allocation problems. The tools were developed for the IDRISI geographic analysis software system and are capable of solving multiple objective land allocation problems with either complementary or conflicting objectives. Jeroen, et al., (2003) address the use of spatial optimization techniques for solving multi-site land-use allocation (MLUA) problems, where MLUA refers to the optimal allocation of multiple sites of different land uses to an area. The problem was solved using four different integer programs (IP), of which three are linear integer programs. The IPs is formulated for a raster-based GIS environment and is designed to minimize development costs and to maximize compactness of the allocated land use. Zielinska, et al., (2008) examined the applicability of spatial optimization as a generative modelling technique for sustainable land-use allocation. The test was specifically on whether spatial optimization can be used to generate number of compromise spatial alternatives that are both feasible and different from each other. The new spatial multiple objective optimization model, encourages efficient utilization of urban space through infill development, compatibility of adjacent land uses, and defensible redevelopment.

2.1 Optimization in Land Use Allocation

Optimization is an activity that aims to finding the best or optimal solution to a problem (Killen, 1983). The task can be expressed using mathematical notation and solved by mathematical programming. In mathematical form, an optimization problem is viewed as finding the values for a set of decision variables $x_1, x_2, \ldots, x_n$ in order to:

$$\begin{aligned} &\text{Optimize:} \quad f(x_1, x_2, \ldots\ldots, x_n) \\ &\text{Such that:} \quad g(x_1, x_2, \ldots\ldots, x_n) \leq \text{ or } = \text{ or } \geq b \\ &\qquad\qquad\qquad \text{For } i = 1, \ldots\ldots, m \end{aligned} \tag{1.0}$$

where $f(x_1, x_2, \ldots\ldots, x_n)$ is some mathematical expression involving ndecision variables, $g(x_1, x_2, \ldots\ldots, x_n)$ for $i = 1, \ldots, m$ represent the left hand sides of the m constraints, and b_i for $i = 1, \ldots, m$ are given fixed variables which occur on the right hand side of the m constraints. The optimization task involves either maximisation or minimisation and the constraints involve equalities or inequalities. In the context of planning, the decision variables, under consideration for example amounts of land to be assigned to various land uses, cannot take negative values.

Therefore, the condition, $x_j \geq 0$ for $j = 1,\ldots,n$ is usually added to the mathematical expression. The optimal solution is values of decision variables $x_1, x_2, \ldots, x_n$ that satisfy the constraints and for which the objective function attains a maximum (or minimum). The optimization problems are not solved analytically but by means of explicit formula. In normal cases, appropriate computational technique (numerical procedures) of optimization is used. Techniques normally used are for example linear and non-linear programming, integer programming, and stochastic or probabilistic programming. In searching for optimal solution, one has to differentiate between a single objective optimization and a multi-objective optimization problem. A single objective optimization usually has a single-valued unique solution. The solution to multi-objective problem is, as a rule, not a particular value, but a set of values of decision variables. For each element in the set, none of the objective functions can be further increased without a decrease of some of the remaining object functions. Every such value of a decision variable in this case is referred as pareto-optimal. There is generally no single optimal solution to the formulation of problems with a collection of objective functions (Goicoecha et al., 1982).

In this paper, we search for a single objective optimization in the context of optimum land use allocation. The optimal solution was land area (regions) that minimised the total development cost. Here, the constraints was represented by total required area, number of regions needed, maximum total land suitability value, minimum total proximity, and minimum total heights. Assignment of decision variables was represented by the feasible regions obtained through an earlier GIS suitability model carried out on a hypothetical problem of determining feasible areas for residential purposes in the state of Penang, Malaysia (Ahamad and Rabiah, 2009; Ahamad et al., 2006). The mathematical programming model for the optimization problem was presented in the form of a single objective function. The decision variables in this case are linear and provide a zero-one solution to the mathematical programming problem. A zero-one solution provides a 'yes or no' answer to an optimal solution, for example 'to select or not to select a particular region'. The most appropriate numerical computation technique was found to be linear integer programming that takes integer zero-one decision variables with linear constraints. The basic concept and understanding of zero-one integer linear programming is explained in the following section.

2.2 Zero-One Linear Integer Programming

The overall structure of a zero-one programming problem as described by Killen (1983) possesses an important property that is absent from the integer programming equivalent. Since each decision variables must equal either zero or one at optimality, therefore the number of possible solutions that will be optimal is limited. Specifically, for a problem involving (n) decision variables, there are (2^n) possible solutions. The search for optimal solution in zero-one programming is in fact the search for combination of zeros and ones for the decision variables for which the objective is optimised whilst maintaining feasibility in terms of the constraints. A common method in dealing with zero-one programming problem is the branch and bound algorithm (Mavrotas and Diakoulaki, 1998). This algorithm uses a combinatorial procedure in its attempt to pursue an efficient search through the possible combinations of integer zero-one solutions. Branch and bound refers to a search process that requires the set of possible solutions to be countable and finite. One disadvantage of this method is that it requires a large number of problems and associated solutions to be stored simultaneously. However, the problem has been solved by employing backtrack programming techniques that obviates the need to retain such large numbers of solutions simultaneously (Christelle et al., 2000).

2.3 Branch and Bound Algorithm

The branch and bound method is a solution strategy in optimization problems. It has ability to eliminate large groups of potential solutions to a problem without explicitly evaluating them. Thesen (1978) described three major reasons in the acceptance of branch and bound algorithms:

1. The method is conceptually simple and easy to understand.
2. The method is easily adaptable to a wide range of different problem situations.
3. The method is well suited for computer implementation.

The branch and bound strategy is based on the premise that the problem to be solved has the properties of combinatorial nature, branchability, rationality, and boundability. These properties are exploited by the branch and bound concept to implicitly and explicitly construct a tree describing all solutions to the problem, and conduct a guided search in this tree for the best solution. To illustrate a simple branch and bound strategy, an example of a zero-one integer programming problem taken from Bunn (1982) is presented.

$$\begin{aligned} &\text{Maximise} \quad Z = x_1 + x_2 + x_3 \\ &\text{Subject to} \quad 3x_1 + 2x_2 + 3x_3 \leq 4 \\ &x_1 + x_2 + 2x_3 \geq 2 \\ &x_i = 0 \text{ or } 1, \text{ for } i = 1,2,3 \qquad (2.0) \end{aligned}$$

The variables x_1, x_2, x_3 can only take on two values, namely 0 or 1, and therefore the number of possible solutions is 2^3. The possible solution to the problem is presented by a sequential branching process, or tree search as shown in Figure 1.

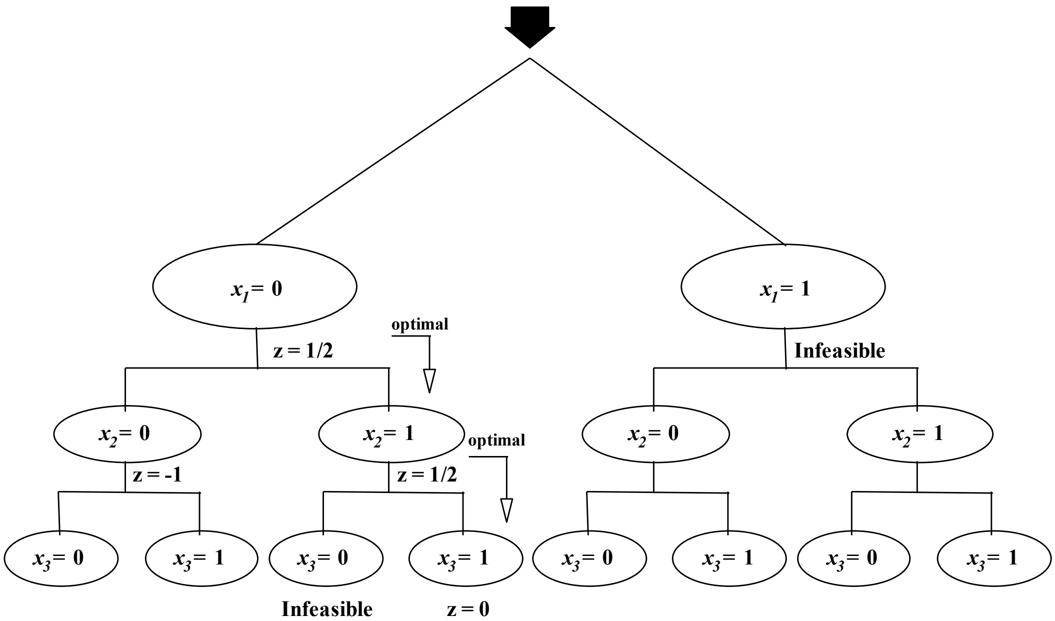

Figure 1. Branching decomposition by tree structure in branch and bound algorithm (Adopted from Bunn, 1982)

Once the underlying structure is developed, the success of the branch and bound algorithm then depends on the extent to which many of the branches do not have to be pursued to their extremities. If a bound on the best attainable solution emerging from one level can be identified and is less than some other best solution already obtained, then that branch need not be investigated further. This bounding principle is fundamental to the economy of the procedure. To demonstrate how the principle works, the zero-one integer problem above is solved for x1 = 0 node and the integer constraints relaxed. When the relaxed problem is solved, the values for $x_2 = 1$, $x_3 = 1/2$, and z = 1/2. The value of z = 1/2 is therefore an upper bound on any solution emanating from this node. As each branch is pursued, more constrained problem is related for which the optimum cannot be greater than the relaxed constraints. The upper bound for $x_1 = 1$ node is similarly computed and the result gives an infeasible solution. Therefore, branches from $x_1 = 0$ node are only considered. For $x_2 = 0$ node, the relaxed problem is solved with $x_1 = 0$, and $x_2 = 0$, which gives z = 1/2 for $x_3 = 1/2$. Since the upper bound for $x_2 = 1$ node is greater than that for $x_2 = 0$ node, this node is therefore pursued. There are two extremities to consider from here, $\{(x_3 = 0) \Rightarrow \text{infeasible}\}$ and $\{(x_3 = 1) \Rightarrow (z = 0)\}$. Hence, the solution $x_1 = 0$, $x_2 = 1$, $x_3 = 1$ gives a higher values (z = 0) than the upper bound for node $x_2 = 0$. Therefore, it is not necessary to backtrack and investigate any further branches. The optimal solution to the original problem is only identified by solving six relaxed sub-problems instead of all eight possible combinations.

In general, branch and bound method commences by fixing a lower bound (maximisation) or upper bound (minimisation) on the optimal solution. In the case of a problem involving many variables, the number of computations required can be reduced considerably if an efficient bound can be determined at the outset or at least early in the calculations (Killen, 1983). Despite potential storage space difficulties which can reach serious proportions in the solution of real world problems, the branch and bound approach has generally proved useful for solving large problems of the type most commonly encountered by geographers and planners. This paper applies the branch and bound algorithm available in MS-Excel Solver to test and evaluate the optimal feasible regions for the residential. Excel solver is an ‘Add-in’ that solves problems related to various methods in Linear Programming. For a given problem, Excel solver can run various permutations and combinations and find out the best possible solution. For comparison purposes, a source code that uses mixed integer branch and bound algorithm in turbo C++ contributed by Eindhoven University of Technology, Netherlands (Notebaert, and Eikland, 2008) is applied consecutively.

3. OPTIMAL LAND ALLOCATION MODEL

A mathematical programming model for a single objective land use allocation is described. The model’s objective was to identify regions that minimised the cost utility while satisfying some specified constraints. The decision variable is represented by the clustered cell configuration defined as feasible regions obtained from GIS suitability model. The problems presented were selection of optimal feasible regions for the development of residential land use.

3.1 Model Formulation

$$\text{Minimise} \quad C_i = \sum_{i=1}^{N} c_i x_i$$

$$\text{Subject to :} \quad \sum_{i=1}^{N} a_i x_i \geq a_{min}; \quad \sum_{i=1}^{N} a_i x_i \leq a_{max}$$

$$\sum_{i=1}^{N} s_i x_i \geq S; \quad \sum_{i=1}^{N} p_i x_i \leq P$$

$$\sum_{i=1}^{N} h_i x_i \leq H; \quad \text{and } x \in (0,1) \qquad (3.0)$$

The land use allocation problem is expressed in its entirety as a binary (0-1) linear integer programming

where $C_i$is the total land development cost, N is the total number of feasible regions, x_i is 1 if allocate region i, and 0 otherwise, c_i is the land cost of feasible region i, $a_i$is thearea of feasible region i, $a_{max}$is the maximum required area, a_{min}isthe minimum required area, $s_i$is the average suitability value of feasible region i, S is the total maximum suitability value required, P is the minimum total proximity achievable, and H is the minimum total average height of selected regions. The objective of the above equation is to minimise the total land development cost of the regions allotted to residential land use. The area constraints limit the total area of regions to be allocated, the proximity and height constraints ensures that the regions selected will be nearest to transportation and lowest in heights, and the suitability constraint maximised the total average suitability value (suitability index) of the regions. The model will identify a set of optimal feasible regions that satisfy the objective function and constraints.

3.2 Model Implementation

The solution approach to the given model is depicted by the flowchart in Figure 2. Spatial database query module called EXTRACT in IDRIS-GIS software was performed to extract numerical attribute values of 42 feasible regions obtained from GIS suitability model inauthor's previous project (Ahamad and Rabiah, 2009)as shown in Figure 3. The module extracts summary statistic of average attribute values from criterion maps used in the suitability model. The attributes are the total land cost per region, average suitability value, average proximity value, average heights, and area of selected regions. These attributes has been considered to minimise the total land development cost of residential land use

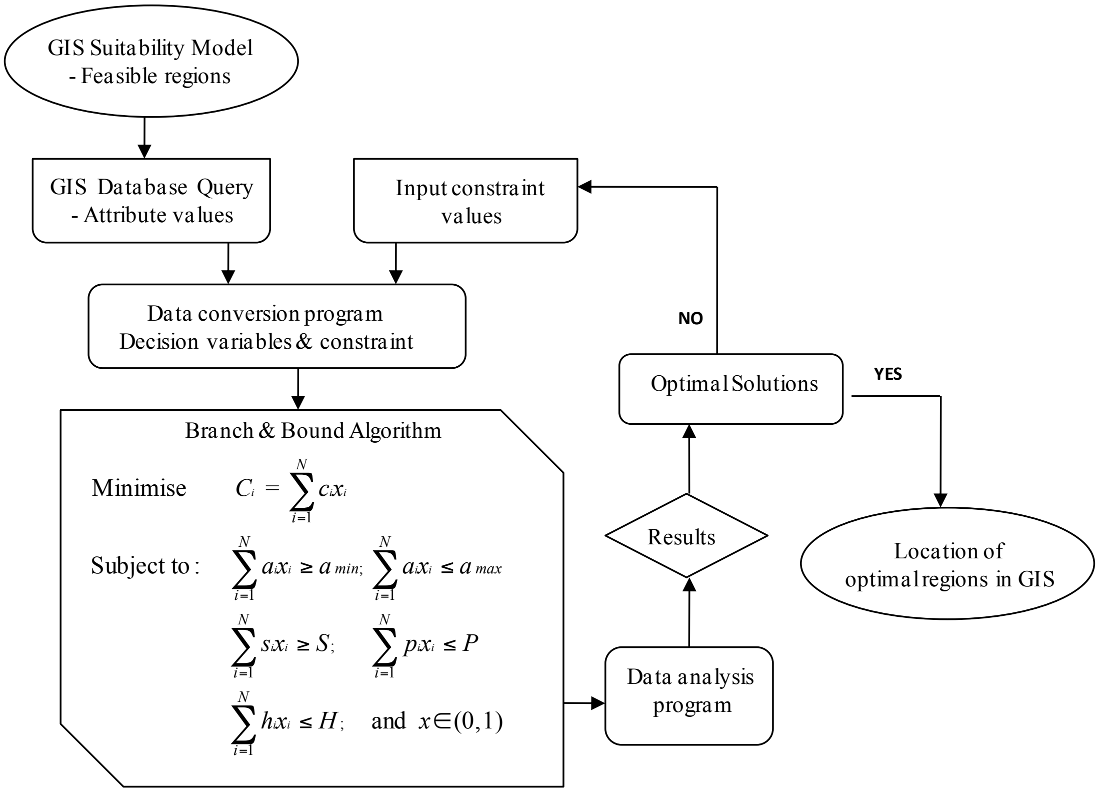

Figure 2. GIS based prescriptive model for solving optimal land use allocation

R	LC	ST	H	P	A
1	5	176	43	364	11
2	39	193	37	341	13
3	36	184	58	392	12
4	36	187	4	297	12
5	8	163	61	235	16
6	46	179	88	606	15
7	59	183	4	507	20
8	14	159	82	320	28
9	7	170	76	842	14
10	7	184	61	1209	14
11	24	170	24	392	48
12	8	168	23	167	16
13	11	154	27	956	21
14	7	151	24	889	14
15	13	168	67	1200	27
16	6	160	270	1144	12
17	8	166	28	440	16
18	9	168	34	765	17
19	11	172	36	448	23
20	30	160	125	357	50
21	31	170	66	325	51

Rgn	LC	ST	H	P	A
22	12	149	168	310	24
23	5	190	49	553	10
24	48	169	95	725	67
25	9	195	76	884	17
26	22	177	20	209	22
27	11	182	97	1005	23
28	28	183	20	311	28
29	7	165	88	608	15
30	7	163	33	138	14
31	6	175	47	556	11
32	15	181	53	294	31
33	5	174	36	352	10
34	10	192	19	180	10
35	26	183	18	169	26
36	12	193	34	210	25
37	10	181	33	458	19
38	8	173	35	394	15
39	13	175	39	164	25
40	7	181	29	169	14
41	38	180	18	358	38
42	7	154	66	1069	13

R-region, LC-land cost, ST-Suitability, H-height, P-Proximity, A-Area

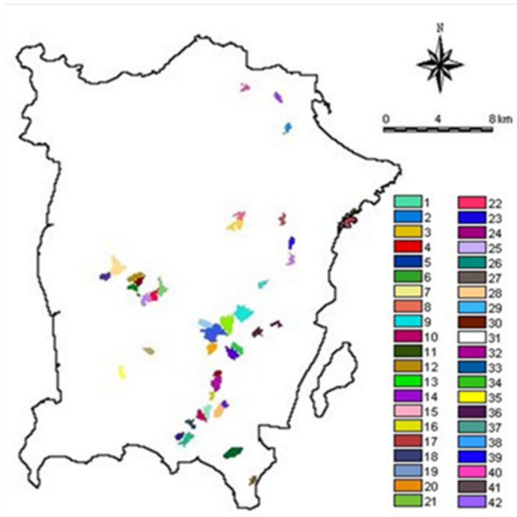

Figure 3. The locational and attributes of 42 feasible regions from GIS suitability model

4. ANALYSIS AND RESULT

A simple data conversion program was developed to convert the ASCII database file to a format readable by the programming algorithm. A series of tests were made in order to determine most optimal solutions for the problem. The problem was set for total area ranging between 350 to 400 hectares and the number of required regions set to 10. Changes were subsequently made to the right-hand values of the constraints to determine optimal solution. The test carried out with the right-hand value constraints set to maximum total suitability value; minimum total proximity; and minimum height of 10 regions. The model searched for optimal solution using the ‘branch and bound’ algorithm. The optimal solution will be 10 regions (decision variables) that produce minimum total land cost at maximised suitability, minimised proximity and minimised heights. The results will give a value 0 or 1 for each region (decision variable). A value 1 indicates that the region selected is optimal and a value 0 is otherwise. It describes the decision variables or regions that are selected in the search for optimal solutions. A data analysis program was subsequently developed to extract the suitability, proximity, and height attributes of the optimal results. It will search and summed the attributes of the optimal regions and display in tabular form. The summary of the results on optimal solution from a series of tests conducted on the zero-one integer programming model using MS-Excel spreadsheetis shown in Table 1. Subsequently, the mixed integer program for branch and bound algorithm contributed by Eindhoven University gives similar result as in Table 2.

Once the optimal feasible regions have been determined, their locations were presented in a map form in GIS software through normal classification process that assigned unique attribute numbers to the selected optimal regions. The most optimal regions represented by 10 decision variables (X_n) with value 1 were obtained from test no. 9 as shown in the table. The model determines optimal solution for the specific condition set, i.e. 10 regions that produced a total area between 350 to 400 hectares. The conditions were set based on the development planning requirement of the local planning authority for 2005-2010. The map location for the ten most optimal feasible regions can be seen in Figure 4. The algorithm search for optimal solution begins with constraint parameters set to the highest suitability score, least total proximity, and least total heights on combination of ten available regions. These constraint's parameters were then gradually increased and decreased to determine optimal value. Optimality in the search is reached when the total suitability constraint is at maximum possible and total proximity and heights are at minimum possible. The objective function of the model presents the total minimum land cost that will occur. Any attempt to increase the suitability constraint and decrease the height and proximity constraints from this point will consequently produce an infeasible solution. The statistical extract of the attributes and the spatial location of ten most optimal regions are described in Figure 4 presenting minimum total land cost as RM2.42 million.

Table 1. Results of zero-one integer programming model (MS EXCEL spread sheet)

Test	SSuitability		SHeight		SProximity		SArea		SCost	10 selected Regions
	Setting	Result	Setting	Result	Setting	Result	Setting	Result		
1	≤1885	1729	≥174	694	≥1935	5048	350 -400	350	198	8,15,19,20,21,24,27,32,36,39
2	≥1730	1734	≤693	605	≤5047	4962	350 -400	350	198	9,11,15,19,20,21,24,32,36,40
3	≥1735	1744	≤694	575	≤4961	4387	350 -400	350	199	11,12,15,20,21,23,24,32,36,39
4	≥1745	1747	≤574	548	≤4386	3737	350 -400	350	199	1,11,19,20,21,24,32,36,37,39
5	≥1748	1752	≤547	534	≤3736	3542	350 -400	353	201	11,19,20,21,24,32,36,37,39,40
6	≥1753	1757	≤533	518	≤3541	3303	350 -400	352	212	11,20,21,24,26,32,36,37,39,40
7	≥1758	1763	≤517	516	≤3302	3263	350 -400	356	216	11,20,21,24,32,35,36,37,39,40
8	≥1764	1764	≤515	486	≤3262	3177	350 -400	350	223	11,12,20,21,24,32,34,36,40,41
9	≥1765	1782	≤485	483	≤3176	3132	350 -400	350	231	11,20,21,24,28,32,34,35,36,40
10	≥1783	1785	≤482	396	≤3131	3117	350 -400	353	242	11,21,24,28,32,35,36,39,40,41
11	≥1786	1786	≤395	395	≤3116	3116	350 - 400	350	239	11,21,24,28,32,35,36,39,40,41
12	≥1786	1786	≥395	487	≤3116	3093	350 -400	351	251	20,21,24,28,32,34,35,36,39,41
13	≤1786	1772	≤395	390	≤3116	3115	350 -400	355	243	11,12,21,24,28,32,35,36,39,41
14	≥1786	1787	≤395	377	≥3116	3162	350 -400	350	251	11,21,24,26,28,32,35,36,40,41

Table 2. Results from mixed integer branch and bound program

Test	Σ Suitability		Σ Height		Σ Distance		Σ Area		10 Selected Regions	Σ Cost
	Setting	Result	Setting	Result	Setting	Result	Setting	Result		
1	≤ 1885	1729	≥ 174	694	≥ 1935	5048	350-400	350	8, 15, 19, 20, 21, 24, 27, 32, 36, 39	198
2	≥ 1730	1731	≤ 693	581	≤ 5047	4363	350-400	350	11, 12, 17, 19, 20, 21, 24, 27, 32, 36	198
3	≥ 1732	1747	≤ 580	548	≤ 4362	3737	350-400	350	1, 11, 19, 20, 21, 24, 32, 36, 37, 39	199
4	≥ 1748	1752	≤ 547	534	≤ 3736	3542	350-400	353	11, 19, 20, 21, 24, 32, 36, 37, 39, 40	201
5	≥ 1753	1757	≤ 533	518	≤ 3541	3303	350-400	352	11, 20, 21, 24, 26, 32, 36, 37, 39, 40	212
6	≥ 1758	1763	≤ 517	516	≤ 3302	3263	350-400	356	11, 20, 21, 24, 32, 35, 36, 37, 39, 40	216
7	≥ 1764	1764	≤ 515	486	≤ 3262	3177	350-400	350	11, 12, 20, 21, 24, 32, 34, 36, 40, 41	223
8	≥ 1765	1782	≤ 485	483	≤ 3176	3132	350-400	350	11, 20, 21, 24, 28, 32, 34, 35, 36, 40	231
9*	≥ 1783	1785*	≤ 482	396*	≤ 3131	3117*	350-400	353*	11, 21, 24, 28, 32, 35, 36, 39, 40, 41	242*
10	≥ 1786	1786	≥ 395	487	≤ 3116	3093	350-400	351	20, 21, 24, 28, 32, 34, 35, 36, 39, 41	251
11	≤ 1786	1772	≤ 395	390	≤ 3116	3115	350-400	355	11, 12, 21, 24, 28, 32, 35, 36, 39, 41	243
12	≥ 1786	1787	≤ 395	377	≥ 3116	3162	350-400	350	11, 21, 24, 26, 28, 32, 35, 36, 40, 41	251

* Optimal solution

Region	Suitability	Heights (m)	Proximity (m)	Area (Hect.)	Land Cost (* 10th Thousand)
11	170	24	392	48	24
21	170	66	325	51	31
24	169	95	725	67	48
28	183	20	311	28	28
32	181	53	294	31	15
35	183	18	169	26	26
36	193	34	210	25	12
39	175	39	164	25	13
40	181	29	169	14	7
41	180	18	358	38	38
Total	**1785**	**396**	**3117**	**353**	**242**

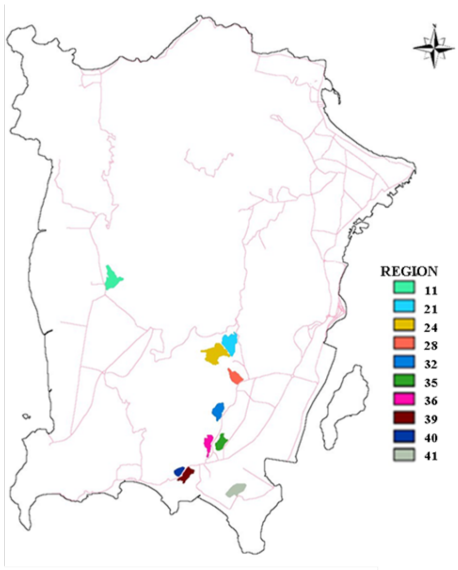

Figure 4. Spatial Location and Attributes of 10 most optimal regions

4.1 Sensitivity of the zero-one integer programming model

The sensitivity test was performed to study the model properties through changes in the input variables and analysed its effect on the model outputs. In land use planning, it is extremely important to study the effect of altering various planning requirements as expressed by constraints in their problem. For a general integer programming problem and its optimal solution, the major types of questions concerning the sensitivity of the solution that may be asked are (Killen, 1983):

1) The effect of changes in the right-hand side value on certain constraints
2) The effect of changes in the coefficients of the objective function
3) The effect of changes in the coefficients of certain constraints values
4) The effect when the constraints are reduced from the original problem

The determination of an optimal solution in the previous section showed sensitivity through changes made to the right-hand side value of suitability, proximity, and height constraints. The effect of the changes does alter some of the selected optimal regions. Therefore, the model is sensitive to the changes made on the constraint values. In land allocation problem, the coefficients of the objective function and constraints have the integer value 1. Therefore, it is not necessary to determine the sensitiveness of the changes in their coefficient values.

The subsequent sensitivity analysis was reduction in the number of constraint where the number of optimal regions required was not considered in the problem. The test was performed for a second time on the database attributes of the 42 feasible regions. The model was tested with the initial right-hand side values previously set on test run no.10 (Table 2). Four difference sets of test were performed to determine the optimal solutions. The results show identical optimal regions in two tests and an increase in the number of regions in the other tests (Table 3). Further test were made on the condition for ‘the total area’ required where it was reduced to half the previous value. Optimal solutions were obtained for four tests as shown in Table 4. The result produces identical optimal regions. This shows that the model output is not sensitive to the reduction of constraint conditions.

The last test summarised the selected optimal regions from the entire test performed on the database attribute values as presented in Table 5. The summary shows a similarity in the optimal regions obtained in cases where the number of region was specified and in cases with unspecified region's condition. In the reduced constraint condition, four out of six optimal regions are similar. This indicates a consistency in the determination of optimal regions (decision variables) by the zero-one integer programming model.

Table 3. Sensitivity test on optimal solutions with unspecified condition on regions

Test	Σ Suitability		Σ Height		Σ Distance		Σ Area		10 Selected Regions	Σ Cost
	Setting	Result	Setting	Result	Setting	Result	Setting	Result		
1*	≥ 1783	1785*	≤ 482	396*	≤ 3131	3117*	350-400	353*	11, 21, 24, 28, 32, 35, 36, 39, 40, 41	242*
2	≥ 1786	2047	≥ 395	732	≤ 3116	3083	350-400	352	5, 8, 11, 20, 21, 22, 30, 32, 35, 36, 39, 40	199
3	≤ 1786	1772	≤ 395	390	≤ 3116	3115	350-400	355	11, 12, 21, 24, 28, 32, 35, 36, 39, 41	243
4	≥ 1786	2110	≤ 395	391	≥ 3116	4252	350-400	351	11, 12, 13, 19, 21, 28 32, 35, 36, 37,39, 41	227

* Optimal solution

Table 4. Sensitivity test on optimal solutions with reduced value for constraints

Test	Σ Suitability		Σ Height		Σ Distance		Σ Area		10 Selected Regions	Σ Cost
	Setting	Result	Setting	Result	Setting	Result	Setting	Result		
1	≥ 893	1059	≤ 198	185	≤ 1558	1550	150-200	151	11, 12, 19, 36, 39, 40	75
2	≥ 1060	1064	≤ 184	169	≤ 1549	1311	150-200	150	11, 12, 26, 36, 39, 40	86
3	≥ 1065	1070	≥ 168	167	≤ 1310	1271	150-200	154	11, 12, 35, 36, 39, 40	90
4*	≤ 1071	1071*	≤ 166	162*	≤ 1270	1248*	150-200	150*	26, 30, 35, 36, 39, 41	118*

* Optimal solution

Table 5. Summary of selected optimal regions

Condition	Area	Selected regions											
Ten optimal regions	242	11	21	24	-	28	-	32	35	36	39	40	41
Unspecified region	242	11	21	24	-	28	-	32	35	36	39	40	41
Reduced constraint	118	-	-	-	26	-	30	-	35	36	39	-	41

5. CONCLUSION

The results show that the model is capable of producing optimal feasible regions based on the objectives and constraints initially set in the allocation problem The model is not sensitive to the number of constraint condition imposed on the problem but is sensitive to the changes made on the value (right-hand side) of the constraints. This indicates that the right-hand side value of the constraints is the determinant in the choice of optimal regions. Therefore the objective function (minimised total land development cost) will be affected by the changes made to the value of any development constraints (land suitability, proximity, or heights). But the reduction of additional constraints may not cause the current optimal solution to any given problem to change. The application of optimization method here shows the importance of choosing a correct value for the constraints.

The study on the ten optimal regions through ground verification has given an indication that their location characteristics are suitable for future residential land allocation. There is no independent evidence from experts (planners or decision makers) that can suggest that the selected regions are the best or to justify the effectiveness of the proposed approach in respect to the residential land use allocation. The main objective is to show that an optimization model can be integrated with GIS/MCE suitability analysis in finding the optimal solution for a specific land use allocation problem. In conclusion, it is important to acknowledge that the decision variables in the optimal problem are all the feasible regions suitable for the residential land use that was prior selected in the earlier suitability analysis model. It does not indicate that the deselected feasible regions are not suitable. They are suitable regions but not the optimal set in terms of the objectives and constraints of the allocation problem.

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