GPS for Orthometric Heights Determination of Long Lines: Egyptian Case Study

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SUMMARY

Long extend engineering projects like highways, canals, railways, are considered to be very essential for developing new urban areas. However, using traditional surveying techniques, for establishing horizontal and vertical surveying datums, has a reverse impact on feasibility studies of such projects. For example, using spirit leveling, for determination of orthometric heights, is cumbersome; time consuming and costly especially in remote areas, where hardly bench marks can be found. On the other hand, GPS leveling technique is assumed to be good alternative for cumbersome traditional techniques of leveling especially in regions where reasonable accuracy is required **(Seeber, 2003)**. Unfortunately, the GPS system produces the heights as ellipsoidal heights, relative to WGS-84 ellipsoid, which are not consistent with orthometric heights. Consequently, in order to convert these ellipsoidal heights to the sought orthometric heights, with accepted accuracy, a suitable geoidal model for the surveyed area has to be known with sufficient reliability for obtaining the needed geoid undulation. The main objective of this study is to evaluate using GPS-leveling technique for determination of orthometric heights of long GPS networks. To achieve such objective, six long GPS networks, located next to national roads in Sinai, Western and Eastern desert, were established. The stations of such networks are first order bench marks, with known orthometric heights. The GPS-leveling technique is used to predict the orthometric heights of such stations. Then, the predicted orthometric heights were compared with the known orthometric heights, to evaluate using GPS-leveling, as alternative technique of traditional spirit leveling. The obtained results show that Most of errors are less 0.3m in areas of gentle slope terrain. Moreover, there is a linear relation between the errors and distances from the reference point. On the other hand, in rough areas, like Sinai, and Red Sea mountains in Ras Gharib-Shiek Fadl, and Edfo-Marsa Alm networks. The accuracy of the predicted orthometric heights is less accurate. Most of errors are about 1.0m. Moreover, there is no linear relation between the errors and distance from the reference point. Also, as alternative methods for determining ellipsoidal heights, Precise Point Positioning (PPP) and relative positioning using international permanent stations were investigated and the results are given.

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1. INTRODUCTION

To evaluate GPS-Leveling technique, a selected GPS stations with known observed orthometric heights are used. The geoidal undulations of these stations are obtained from a suitable geoidal model of area of investigation. Then, the technique of GPS-Leveling is used to convert the ellipsoidal heights of these stations to the corresponding orthometric heights. Finally, the predicted orthometric heights are compared with the corresponding known observed orthometric heights. This will be done on relative basis, that is, dealing with orthometric heights difference instead of the absolute ones. This leads to optimum accuracy, since most of the systematic errors, inherently existing in both the derived geoidal model and ellipsoidal heights, will be eliminated or at least minimized, as mentioned earlier. The obtained discrepancies between the predicted orthometric heights and the known observed orthometric heights are used to test the feasibility of using GPS-Leveling technique, for some related practical applications.

2. BASIC CONCEPT OF USING GPS FOR ORTHOMETRIC HEIGHT DETERMINATION

Once the ellipsoid height (h) is provided from GPS data for points, whose orthometric height are required to be determined, and the geoid undulation N at these points are available from any pre-determined geoidal surface. The corresponding orthometric height (H) can be calculated using the following equation:

$$\mathbf{H = h - N} \quad (1)$$

However, the accuracy of the resulted orthometeric heights, by the previous equation, is affected by the inherently existing biases in its both components namely the ellipsoidal heights (**h**) and geoidal undulation (**N**).

Consequently, by adopting relative approach, many systematic errors, that common to nearby stations **i** and **j**, are greatly reduced by adopting the relative differential case. Accordingly, the difference in orthometric height between the two stations **i** and **j**, **ΔHij**, can be obtained with higher accuracy than the absolute values **Hi** and **Hj (Issa, 2000).** that:

$$\mathbf{\Delta Hij = \Delta hij - \Delta Nij} \quad (2)$$

Then, the orthometric height of point **j, Hj,** is computed as follows:

$$\mathbf{Hj = Hi + \Delta Hij = Hi + \Delta hij - \Delta Nij} \quad (3)$$

Where, Hj = the sought orthometric height at point j
Hi = the known orthometric height at point i

$$\Delta hij = hj - hi \quad (4)$$

$$\Delta Nij = Nj - Ni \quad (5)$$

It is clear that, the subtraction of equation (4) will cancel out the common biases of ellipsoidal heights, while, the subtraction of equation (5) will cancel out the common biases of geoidal undulations, since the biases are assumed to be the same for the two nearby stations.

3. OBTAINING GEOIDAL UNDULATIONS N OF NETWORKS

The geoid undulation at each station can be obtained from two sources; first, from one of earth geopotential models like EGM96 through the Internet, second, from any national gravimetric geoid. In this paper, ASU2005 gravimetric geoid of Egypt is used.

3.1 ASU2005 National Gravimetric Geoid of Egypt

Many researchers have tried to determine the geoid on global basis for global geopotential models; however, the results will give only the long wavelength feature of the geoid **[Mogahed, 1999].** Accordingly, a local geoidal model for the area of interest (such as the Egyptian territory) based on all gravity data should be computed and updated periodically to get best resulted shortwave length feature of the geoid that fits the current geodetic needs **[El-Tokhey, 1993].** Accordingly, a gravimetric geoid for Egypt using the Fast Fourier Transformation Technique (**FFT**) was determined by Ain Shams University in 2005 and named ASU2005, relative to the WGS-84 datum [**Mogahed, 2006].** The geoidal undulations covers all the area of Egypt each 3` x 3 grid so that the geoidal undulation of any point can be obtained by interpolation using latitude and longitude of such point.

4. OBTAINING ELLIPSOIDAL HEIGHTS OF NETWORKS

Three GPS positioning techniques to determine ellipsoid heights of any unknown point are investigated in this paper.

1. The technique of Precise Point Positioning (PPP).
2. The technique of solving such point relative to one or more of international permanent tracking stations.
3. Relative positioning technique using known reference point.

The first and second methods will be investigated in sections 4.1 and 4.2.

4.1 Solution Using Precise Point Positioning (PPP)

This is a powerful strategy for estimating the coordinates and elevation of a single station using precise ephemeris, which can be taken from different agencies like International GNSS Service (IGS) (**Seeber, 2003**). The IGS precise ephemeris can be downloaded from interactive GNSS calendar website **at** http://www.rvdi.com/freebies/gpscalendar.html .

4.2 Relative Solution to International Permanent Tracking Stations

Figure (1) shows the permanent tracking stations of IGS which are close to Egypt. Two of them, ADIS and RAMO, are used in this paper. On the Other hand, **Figure (2)** shows the permanent stations of European Reference Frame (EUREF), and considered to the most nearest tracking stations to Egypt, where two of them, TUC2 and NICO, are also used in this paper. **Table (1)** demonstrates the main characteristics of four stations.

Figure (1): IGS tracking stations close to Egypt.

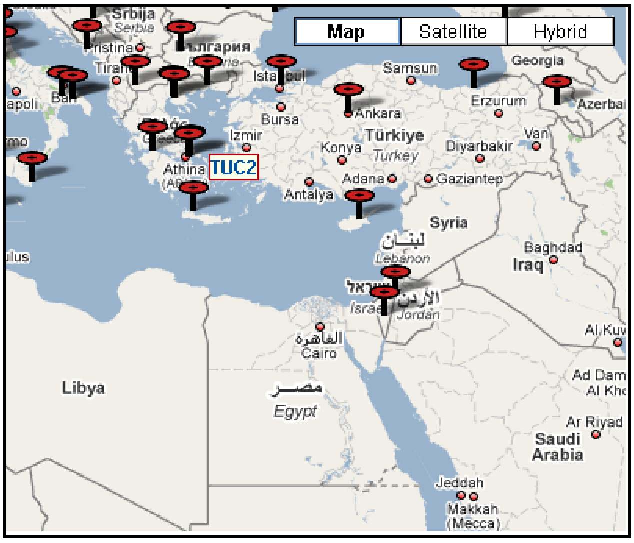


Figure (2): EUREF tracking stations close to Egypt.

Figure (**3**): Fixed point (**K**) in Nasr city, Cairo.

Table (1): The main characteristics of the used permanent stations.

	ADIS	RAMO	TUC2	NICO
Location	at the Addis Ababa University premises in Ethiopia	sedimentary bedrock in Mitze Ramon, Israel	sandstones and cemented clay stone bedrock in Chania, Greece	sandstones and cemented clay stone bedrock in Nicosia, Cyprus,
The monument	a pillar of height 1.5 m, and its foundation is a concrete block of 1 m depth.	a brass nail, and its foundation is a steel rods and concrete block where the ashtech	a pillar of height 1.5 m, and its foundation is a concrete block of 1 m depth.	a pillar of height 1.5 m, and its foundation is a concrete block of 1 m depth.

		antenna is drilled.		
Receiver	Trimble	(ashtech) rack.	The rogue SNR-8000	The rogue SNR-8000
Delivering data	● Daily RINEX files (30 sec. sampling) ● Hourly RINEX files (30 sec. sampling) ● Real-Time NTRIP stream with RTCM 3.0.	● Daily RINEX files (30 sec. sampling)	● Daily RINEX files (30 sec. sampling) ● Hourly RINEX files (30 sec. sampling)	● Daily RINEX files (30 sec. sampling). ● Hourly RINEX files (30 sec. sampling). ● Real-Time of format: RTCM 3.0, free access.
Coordinates	Latitude: 09° 02' 6.5" N Longitude: 38° 45' 58.7" E Ellipsoidal Height: 2439.137(m)	Latitude: 30° 35' 51.4"N Longitude: 34° 45' 47.3"E Ellipsoidal Heights: 886.829(m)	Latitude: 35° 31' 59.5"N Longitude: 24° 04' 14.0"E Ellipsoidal Height: 160.943(m)	Latitude: 35° 08' 27.5" N Longitude: 33° 23' 47.2" E Ellipsoidal Heights: 190.077 (m)

To be able to compare the first and second techniques an experiment has been done. Fixed reference point (**K**), has been fixed in Nasr city, Cairo, **Figure (3)**. A static GPS observations have been collected at such point (**K**) using a dual frequency receiver of LEICA system 500 for different period sessions of 0.5 hr, 1.0 hr, 2.0 hr, 4.0 hr, 5.5 hr, 12.0 hr, 24.0 hr.

To obtain ellipsoidal heights, the WGS84 coordinates of such point (**K**) were obtained by using two techniques of point positioning: the first technique of stand alone single point positioning with precise ephemeris strategy of data post processing, the so called precise point positioning (**PPP**). Whereas, the second technique is solving such point relative to the four permanent tracking stations close to Egypt; **ADIS**, **RAMO**, **TUC2**, and **NICO** tracking stations.

Relative position is possible if two receivers are used simultaneously to collect (code or carrier phase) measurements. Normally, the coordinates of one site is known and the position of the other point is determined relatively to the known point. However, for remote areas where there is no known bench mark or surveying points. The Reference data of one or more of permanent reference stations can accessed via internet as RINEX format, for the same days of data collection, then, it can be imported to processing software. On the other side, the raw static observation data of unknown point can be imported to be processed relatively to reference data of permanent reference stations.

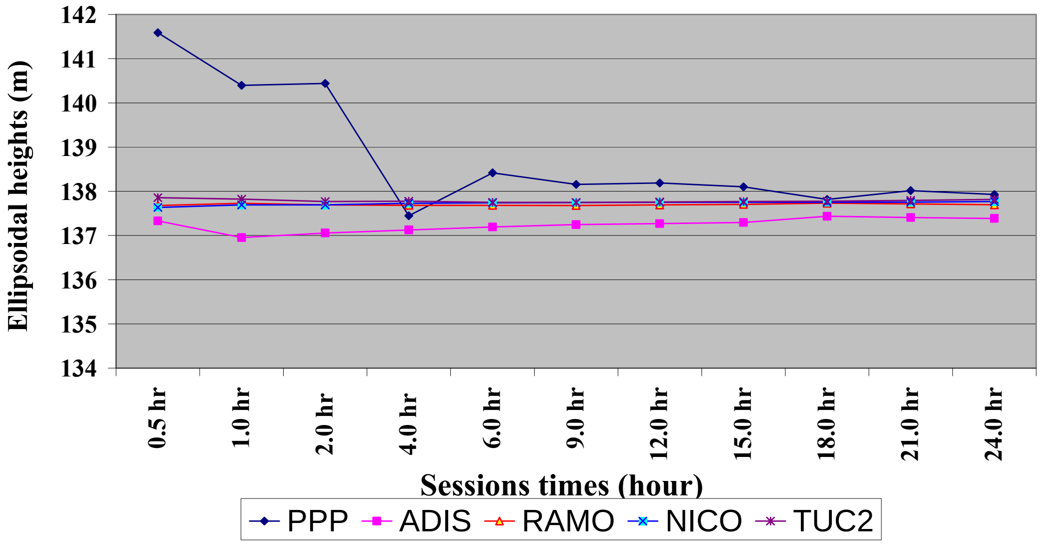
In our case study, the raw data of unknown point (**k**), with different sessions, was imported to

LEICA Geo Office Combined version 5, then, RINEX data, of 24 hours of day of data collection, of ADIS, RAMO, TUC2, and NICO were downloaded via internet and imported to processing software. Then, the raw data of point (**k**) were processed relatively to the four points. Since our main interest is ellipsoidal heights, a comparison between the obtained ellipsoidal heights, for different sessions times, corresponding to each one of the four permanent reference stations and also ellipsoidal heights obtained by PPP technique can be seen in **Table (2)**, while **Figure (4)** demonstrate the comparison graphically.

Giving a close view on the **Figure (4)**, it can be easily found that: the four curves of relative positioning to **ADIS**, **RAMO**, **NICO**, and **TUC2** are approximately horizontal, which means that length of session time is not important. while PPP curve became approximately horizontal after four hours sessions time.

Table (2): Ellipsoidal heights in meters obtained for different sessions times relative to the four permanent reference stations and the ellipsoidal heights obtained by PPP technique.

	0.5 hr	1.0 hr	2.0 hr	4.0 hr	6.0 hr	9.0 hr	12.0 hr	15.0 hr	18.0 hr	21.0 hr	24.0 hr
PPP	141.59	140.39	140.44	137.45	138.42	138.16	138.19	138.1	137.82	138.02	137.93
ADIS	137.33	136.95	137.06	137.13	137.19	137.25	137.27	137.29	137.44	137.41	137.39
RAMO	137.68	137.73	137.69	137.68	137.69	137.68	137.69	137.7	137.73	137.71	137.69
NICO	137.63	137.69	137.69	137.73	137.74	137.75	137.75	137.75	137.75	137.75	137.77
TUC2	137.86	137.82	137.77	137.77	137.75	137.75	137.76	137.77	137.78	137.8	137.82



Figure(4): Ellipsoidal heights of point (k) determined by PPP technique and relative psitioning to ADIS, RAMO, NICO, and TUC2.

5. DESCRIPTION OF USED LONG GPS NETWORKS

For this study, six different long GPS networks are observed and are used in investigating the feasibility of GPS- Leveling technique. These six GPS networks are located in Egypt remote areas like Sinai, Eastern and Western desert, where the important new cities and large projects are supposed to be constructed. The location of these networks is illustrated in **Figure (5)**. Each one of the six long shape GPS networks were constructed by occupying and observing Egyptian Surveying Authority (ESA), national first order bench marks, as follow:

- Stations of network number (1), are bench marks located besides desert roads of middle of Sinai (Sadr El-Hetaan- Yalk mountain),
- Stations of network number (2), are bench marks located besides oasis road (El-Giza- EL-Bawity Oasis),
- Stations of network number (3), are bench marks located besides (Ras Gharib-El-Sheik Fadl) road in the Eastern desert.
- Stations of network number (4), are bench marks located besides (Edfo - Marsa Alm) road in the Eastern desert.
- Stations of network number (5), are bench marks located besides (Asyut-Kharga) road in the Western desert,
- Stations of network number (6), are bench marks located besides (Kharga-Paris oasis) road, in the Western desert,

The main characteristics of these GPS networks are shown in **Table (3)**.

Table (3): **The main characteristics of six long GPS networks.**

Network	Length (Km)	Number of Stations	Known Orth. Heights Variation		EGM96 Geoidal Undul. Variation		ASU2005 Geoidal Undul. Variation	
			Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
Sinai	108.5	18	181.99m	486.534m	16.18m	18.06m	16.32m	18.31m
Ras Gharib-Shiek Fadl	39.35	7	133.89m	548.18m	12.8m	14.7m	13.45m	13.87m
Edfo- Marsa Alm	158.53	19	153.8m	222.54m	11.93m	13.25m	10.69m	12.50m
Giza-Bawitty Oasis	122.75	27	210.28m	316.47m	16.04m	16.60m	14.65m	14.88m
Asyut-harga Oasis	193.25	11	27.53m	69.10m	12.66m	13.05m	12.44m	12.73m
Kharga-Paris Oasis	54.9	9	43.16m	69.10m	12.7m	12.73m	12.65m	12.86m

5.1 Methodology of Investigation of Long Shape Networks

The methodology of investigating of long shape GPS networks to predict the orthometric heights difference from equations (3) and (4) will be as follow:

- Different GPS campaigns have been performed to obtain WGS84 coordinates and ellipsoidal heights of bench marks of the six long shape GPS networks.

Figure (5): The location of the six GPS networks.

- The geoidal undulations of the stations of all used networks were obtained from two sources; EGM96 and ASU2005. Consequently, the calculations for obtaining orthometric heights will be performed twice; first using EGM96 geoidal undulations and the second using ASU2005 geoidal undulations.
- The calculations are performed by using suitable reference station (i), with known ellipsoidal height, orthometric height, and geoidal undulation. While, station (j) represent the remaining network stations whose predicted othometric heights are required.

- The reference station (i) was chosen at the edge of the test network, to investigate the relation between the length of baseline and the resulted error.
- The predicted orthometric heights of two cases of calculations, using EGM96 undulations and ASU2005 undulations, are compared with the known observed orthometric heights of stations, to obtain the errors, then; the maximum, minimum and average absolute errors are given.
- A comparison between the variations of errors with distances of used EGM96 geoidal undulations and those of ASU2005, is graphically demonstrated, to show the effect of geoidal undulation on the predicted orthometric heights.

5.2 Presentation and Analysis of Obtained Results

According to the obtained results of using GPS-leveling technique for predicting the orthometric heights of six long shape GPS networks and the statistics, which are shown in **Table (4)** . The following remarks can be concluded:

a. The accuracy of the predicted orthometric heights is improved considerably in areas of gentle slope terrain, like Oasis road, Asyut-Kharga, and Kharga-Paris networks. As it can be seen that most of errors are less 0.3m.

b. The accuracy of the predicted orthometric heights is getting worse considerably in areas of rough slope terrain, like Sinai, and Red Sea mountains in Ras Gharib-Shiek Fadl, and Edfo-Marsa Alm networks. As it can be considered that errors could be reached more than 1.0 m, with increasing the distance from reference point.

c. In general, the errors of the predicted orthometric heights, is increased with the increasing the distance from the reference point.

Table (4): Results of using relative GPS-leveling technique for six long shape networks.

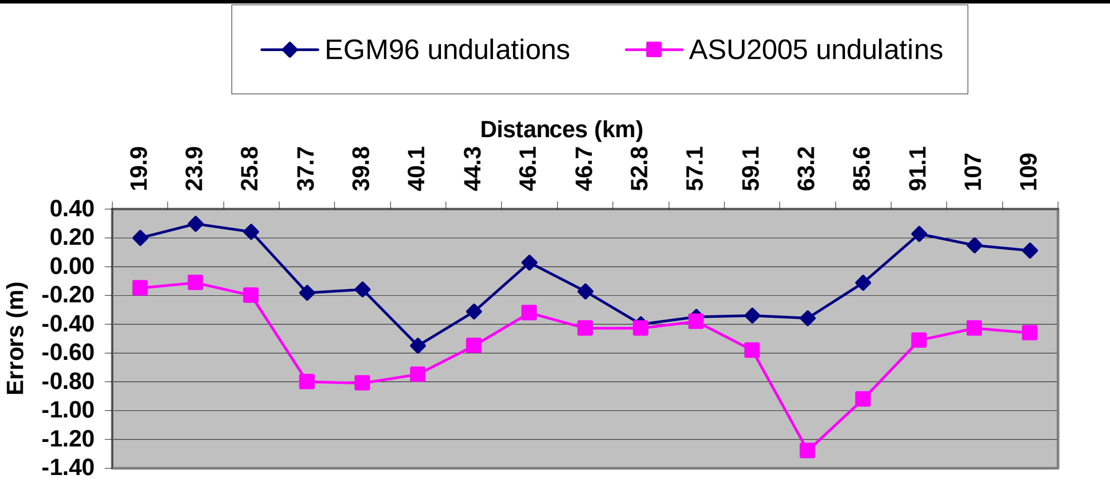
Network	Distance	Number of stations	Statistics	Reference point	
				EGM96	ASU2005
Sinai	108.5	18	Min. Absolute error	0.03m	0.11m
			Max. Absolute error	0.55m	1.28m
			Average Absolute error	0.25m	0.54m
Ras Gharib-Shiek Fadl	39.35	7	Min. Absolute error	0.05m	0.31m
			Max. Absolute error	0.22m	2.0m
			Average Absolute error	0.11m	1.17m
Edfo-Marsa Alam	148.1	19	Min. Absolute error	0.0m	0.01m
			Max. Absolute error	1.74m	1.20m
			Average Absolute error	0.47m	0.48m
Giza-Bawitty Oasis Road	122.78	27	Min. Absolute error	0.03m	0.03m
			Max. Absolute error	1.3m	0.76m
			Average Absolute error	0.57m	0.39m
Asyut-Kharga Oasis	193.25	11	Min. Absolute error	0.02m	0.01m
			Max. Absolute error	0.15m	0.14m
			Average Absolute error	0.08m	0.07m
Kharga-Paris Oasis	54.9	9	Min. Absolute error	0.07m	0.05m
			Max. Absolute error	0.24m	0.24m
			Average Absolute error	0.19m	0.18m

5.3 Variation of Errors with Distances

The variations of errors with distances of used EGM96 geoidal undulations and those of ASU2005 are graphically demonstrated, to show the effect of geoidal undulation on the predicted orthometric heights, as shown in the following sections:

5.3.1 Sinai Network

Figure (6) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is no linear relation between variation of error and distances from edge reference point, for both EGM96 and ASU2005 undulations. This may be due disturbance of geoid in Sinai area.



Figure(6): Variations of errors with distances according to EGM96 and ASU2005 undulations, for Sinai network.

5.3.2 Oasis Road (Giza-Bawitty) Gps Network

Figure (7) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is linear relation, between variation of error and distances from edge reference point, for both EGM96 and ASU2005 undulations. This may be due gentle smooth of geoid in that area. However, there is a jump of errors, starts with error of 40 cm to error of about 80 cm, for set of middle stations. To investigate the reason of this jump, the orthometric heights of this set of stations were observed another time, and it was found that there are no errors of the observed orthometric heights. Consequently, the reason of this jump is expected to be from errors of ellipsoidal heights or geodidal undulations of stations of this set.

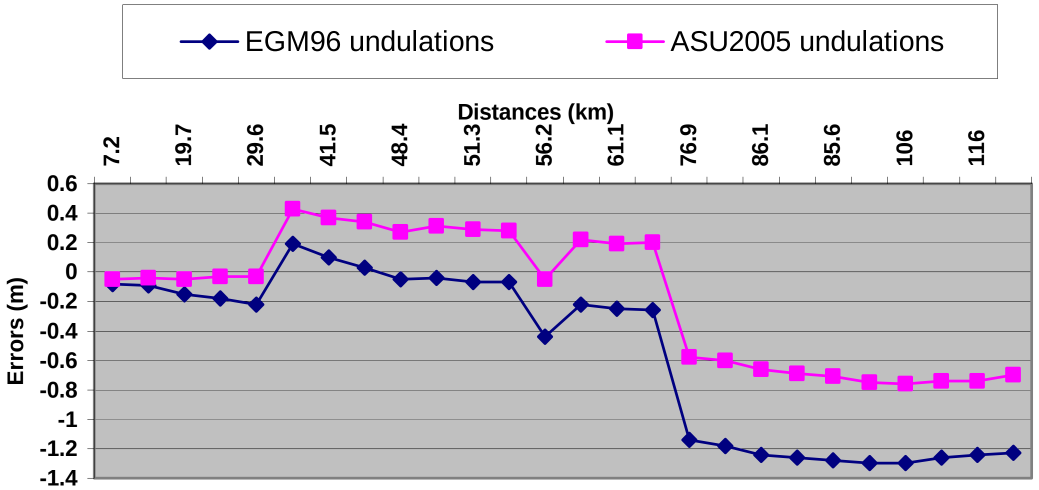
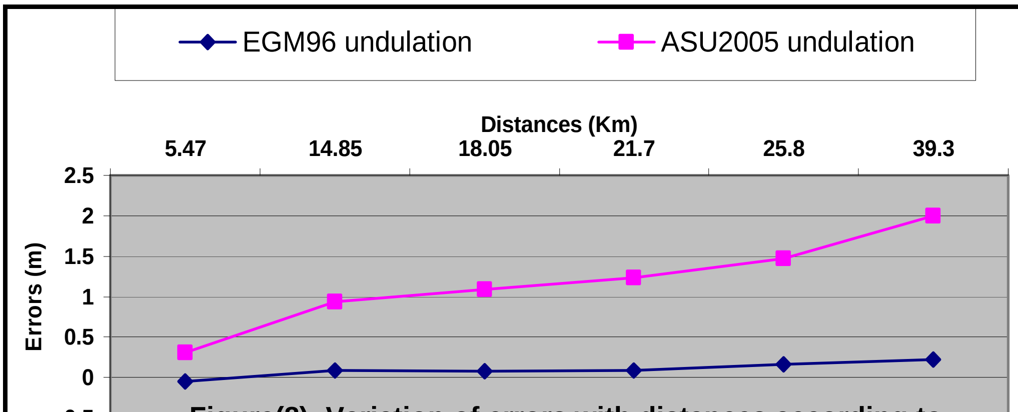


Figure (7): Variation of errors with distances, according to EGM96 and ASU2005 undulations, for Giza-Bawitty road network.

5.3.3 Ras Garib- El-Sheik Fadl Network

Figure (8) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is linear relation, between variation of error and distances from edge reference point, for both EGM96 and ASU2005 undulations.



Figure(8): Variation of errors with distances according to EGM96 an ASU2005 undulations, for Ras Gharib- El-Sheik Fadl network.

5.3.4 Edfo- Marsa Alm Network

Figure (9) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is linear relation in the first 33Km, between variation of error and distances from edge reference point, for both EGM96 and ASU2005 undulations. Then, there is no linear relation, for the rest distance.

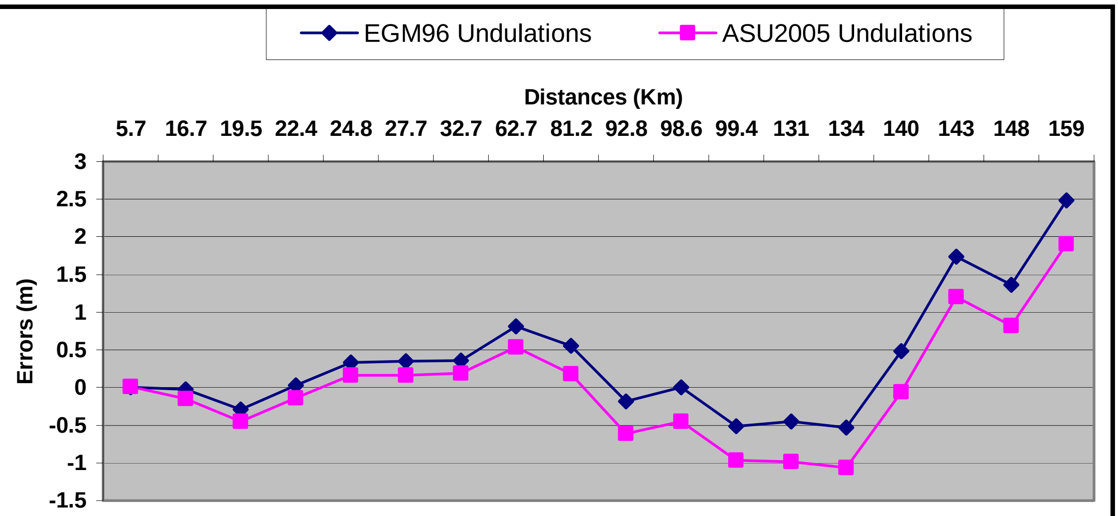


Figure (9):Variations of errors with distances according to EGM96 and ASU2005 undulations, for Edfo-Marsa Alm network.

5.3.5 Asyut- Kharga Network

Figure (10) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is linear relation between variation of error and distances from edge reference point.

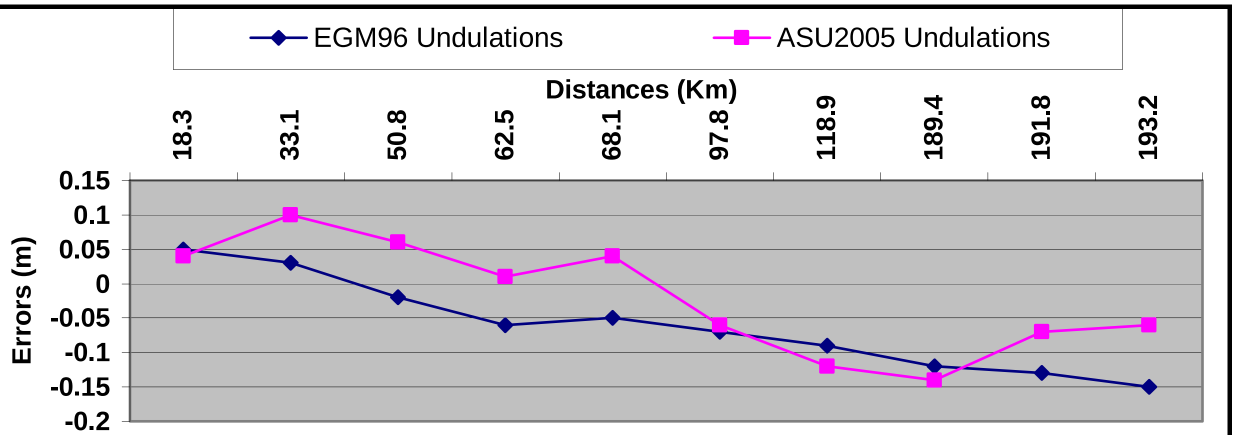


Figure (10): Variation of errors with distances, according to EGM96 and ASU2005 undulations, for Asyut-Kharga network.

5.3.6 Kharga- Paris Oasis Network

Figure (11) shows comparison between variation of errors with distances according to EM96 and ASU2005 undulations, where, it can be seen that there is linear relation between variation of error and distances from edge reference point, for both EGM96 andASU2005 undulations.

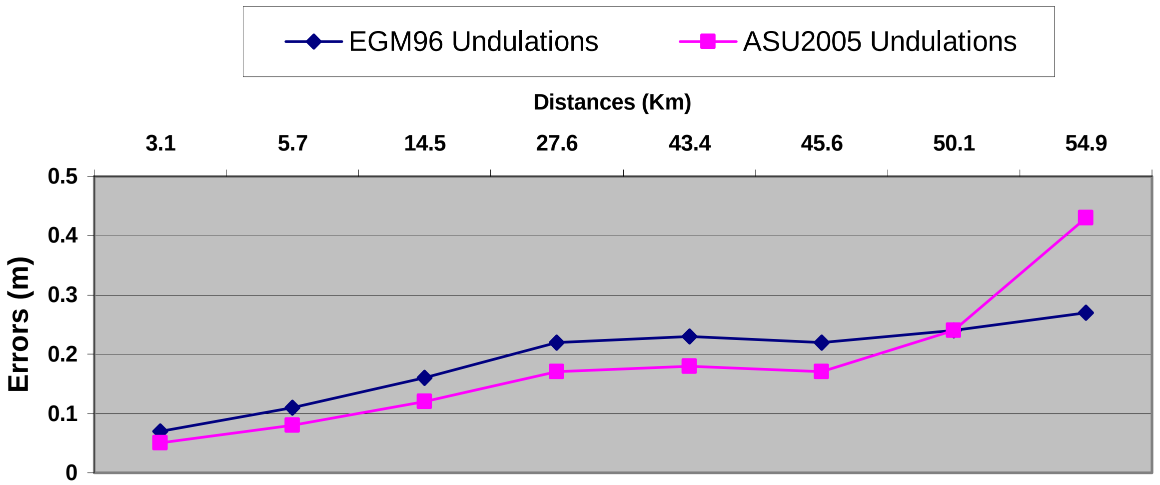


Figure (11): Variation of errors with distances, according to EGM96 and ASU2005 undulations, for Kharga-Paris network.

6. CONCLUSIONS

In areas of gentle slope terrain, like Oasis road, Asyut-Kharga, and Kharga-Paris networks, the accuracy of the predicted orthometric heights, is reasonable. Most of errors are less 0.3m. Moreover, there is a linear relation between the errors and distances from the reference point. On the other hand, in rough areas, like Sinai, and Red Sea mountains in Ras Gharib-Shiek Fadl, and Edfo-Marsa Alm networks, the accuracy of the predicted orthometric heights is less accurate. Most of errors are about 1.0m. Moreover, the relation between the errors and distance from the reference point is not linear.

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