Modeling of Engineering Structures Displacement by Using the Euler Method

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Key words: displacement, Euler method, subspace identification, dynamic system

SUMMARY

The Euler Method is adapted for use in modeling geodetic deformation. Using the subspace identification method parameters of the model has been estimated here. The results indicate an acceptable accuracy of the subsidence prediction and the velocity of subsidence by this method.

SAŽETAK

Ojlerova metoda je prilagođena za modelovanje geodetskih deformacija. Primenujući identifikacionu metodu podprostora ocenjivani su parametri modela. Rezultati pokazuju zadovoljavajuću tačnost predikcije sleganja i brzine sleganja ovom metodom.

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1. INTRODUCTION

The construction project realization is based on certain assumptions and simplifications. The result of this is that civil design and inference include unproved assumptions, leading to the over-dimensioning of some strucutre elements, which makes the building more expensive.
It is known that each structure is subject to displacements and deformations. They appear from external and internal influences such as wind force, changes in temperature, tectonic and seismology impact, changes of underground water level, and static and dynamic loads. These deformations appear as: deflection, slope, turn, distort of the structure, and damages in the form of cracks and significant breakages are also possible.
The displacement and deformation measurements are necessary for the estimation of the structure's condition, while estimation of the object's condition is connected with building, rebuilding, or with structure reinforcing, as well as the checking of the theoretical assumptions.
The deformations of a structure are defined as a result of a process. The best way of modeling structures' behavior as a function of time and other influences is based on observations – the system identifications, because the computations are founded on assumptions and simplifications. The system identification is a type of modeling based on an experiment, i.e. the measurements conducted upon real structures or prototypes. It is a part of the mathematical model and it is expressed as a form of a transfer function, or a state-space model. The exact model of a dynamic process is rarely known. Commonly, a dynamic system is approximated with an applicable model, which approximates the real system with satisfying accuracy.
In this paper, The Euler method is used for the approximation of a dynamic system, whereas for the system identification parameters, using the Euler method, the subspace method is used.

2. THE EULER METHOD

The Euler method is a first order numerical procedure for solving ordinary differential equations with a given initial value. The integration of the dynamical system model by the Euler method will be explained on a simple example of a first order aperiodic system. Also, the structure subsidence can be described by using a first order system. The transfer function of the aperiodic system is

$$G(s)=\frac{X(s)}{M(s)}=\frac{a}{s+a} \tag{2.1}$$

where:
$X(s)$ is a Laplace transformation of an output signal,
$M(s)$ is a Laplace transformation of an input signal.

The system differential equation is

$$\dot{x}(t) + ax(t) = am(t)\,, \tag{2.2}$$

and an appropriate integral equation is

$$x(t) = \int_{-\infty}^{t} \left[-ax(\tau) + am(\tau)\right] d\tau\,. \tag{2.3}$$

A response of the system at the moment of selection $t = kt$ on an exciting signal $m(t)$ with an initial value $x(0)$ can be expressed by

$$\begin{aligned} x(kT) &= \int_{-\infty}^{kT-T} \left[-ax(\tau) + am(\tau)\right] d\tau + \int_{kT-T}^{T} \left[-ax(\tau) + am(\tau)\right] d\tau \\ &= x(kT-T) + \int_{kT-T}^{T} \left[-ax(\tau) + am(\tau)\right] d\tau \end{aligned} \tag{2.4}$$

The first term on the right side of the previous equation is a value of the response on the instant $t = kT - T$, and the second term represents the area, which is in the range $kT - T \le t < kT$, above bounded by the function $-ax(t) + am(t)$ and below by the time axis (Figure 2.1).

There are many procedures for the determination of the area value. The simplest one is the Euler method, where the approximation of the area is a parallelogram with a span T and a height which is equal to the value of the subintegral function at point $t = kT - T$. If we adopt that the height is $-ax(kT - T) + am(kT - T)$, then the value of the response $x(t)$ at the moment $t = kT$ is approximated as

$$\begin{aligned} x(kT) &= x(kT-T) + T\left[-ax(kT-T) + am(kT-T)\right] \\ &= (1 - aT)x(kT-T) + Tam(kT-T) \end{aligned}\,. \tag{2.5}$$

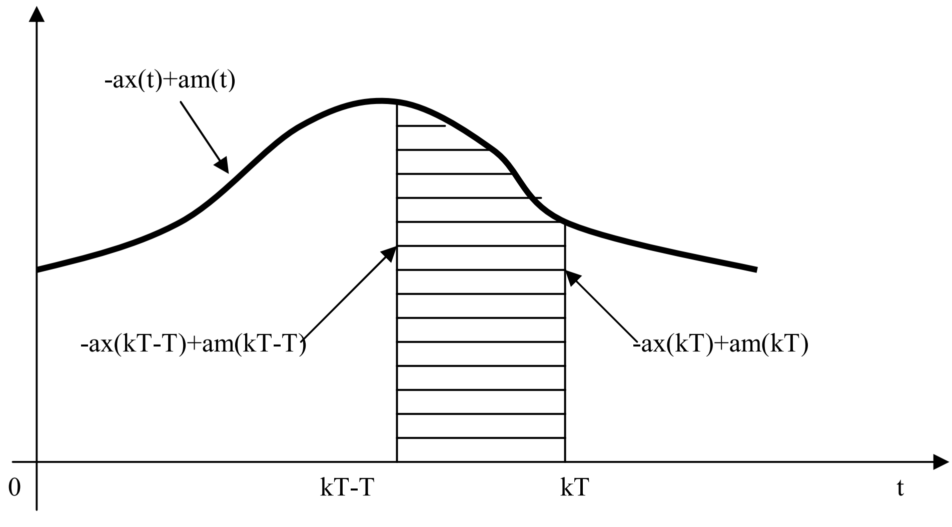


Figure 2.1. Illustration of the numerical integration process

2.1 Subspace method

A linear system can be represent in state space form:

$$\begin{aligned}\mathbf{x}(kT+T) &= \mathbf{A}\mathbf{x}(kT)+\mathbf{B}\mathbf{u}(kT)+\mathbf{w}(t)\\ \mathbf{y}(kT) &= \mathbf{C}\mathbf{x}(kT)+\mathbf{D}\mathbf{u}(kT)+\mathbf{v}(kT)\end{aligned} \tag{2.6}$$

where:
$\mathbf{x}$ is a $n-$dimensional state vector
$\mathbf{u}$ is a n_u-dimensional input vector
$\mathbf{y}$ is a n_y-dimensional output vector
$\mathbf{v}$ is a n_y-dimensional noise vector
$\mathbf{w}$ is a $n-$dimensional process noise vector
A, B, C and **D** are parameter matrices of the appropriate dimension.

The main idea behind subspace identification technique is that given the input-output data sequence $\mathbf{u}(kT)$, $\mathbf{y}(kT)$, $k=1,2,...,N$,the state sequence $\mathbf{x}(kT)$, $k=1,2,...,N$ **,** is estimated first, and then the state space matrices **A, B, C** and **D** are found using a least squares procedure.

Assume for a moment that not only **u** and **y** are measured, but also the state vector **x**. We can form a linear regression form the state space model with know **u**, **y** and **x**:

$$\mathbf{Y}(kT)=\begin{bmatrix}\mathbf{x}(kT+T)\\ \mathbf{y}(kT)\end{bmatrix},\quad \Theta=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}$$

$$\Phi(kT)=\begin{bmatrix}\mathbf{x}(kT)\\ \mathbf{u}(kT)\end{bmatrix},\quad \mathbf{E}(kT)=\begin{bmatrix}\mathbf{w}(kT)\\ \mathbf{v}(kT)\end{bmatrix}.$$

Then the state space model (2.6) may be written as:

$$\mathbf{Y}(kT)=\Theta\Phi(kT)+\mathbf{E}(kT)\,. \tag{2.7}$$

From this equation, the matrix elements in Θ can be estimated by the simple least squares method.

3. DESCRIPTION OF THE STRUCTURES DISPLACEMENT

The main equation in Civil Engineering practice is the equation of motion:

$$M\ddot{x}+C\dot{x}+Kx=F \tag{3.1}$$

where M is the mass, K is the stiffness and C is damping coefficient. x is displacment response of the structure to some extrenal force F, where the dots above the displacment variable are indicative of the derivation order.
On the basis of the Euler method we will determine an approximate discrete model of a continuous system described by the equation (3.1). First, the model is translated into the state-space differential equations.

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\frac{C}{M}x_2(t) - \frac{K}{M}x_1(t) + \frac{F}{M} = -ax_2(t) - bx_1(t) + m \quad (3.2)$$

The corresponding state-space differential equations obtained by using the Euler method are:

$$\begin{bmatrix} x_1\left[(k+1)T\right] \\ x_2\left[(k+1)T\right] \end{bmatrix} = \begin{bmatrix} 1 & T \\ -bT & 1-aT \end{bmatrix} \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} + \begin{bmatrix} T \\ 0 \end{bmatrix} m, \quad (3.3)$$

i.e. in the form of (2.6) (first equation)

$$\mathbf{x}(kT+T) = \mathbf{A}\mathbf{x}(kT) + \mathbf{B}\mathbf{u}(kT) \quad (3.4)$$

$x_1(kT)$ represents the displacement of a benchmark at the moment *kT*, and $x_2(kT)$ represents the velocity of the benchmark displacement.

With the geodetic monitoring of the structures we collect the output signals of the dynamic system. Because of that our data belong to the time series. Unfortunately during the geodetic monitoring we are rarely familiar with the external forces that influence the structure. If the forces are not known, then the last term on the right side of the equation (3.3) does not exist. When we use the Euler method (3.1), we assume that the structure of the parameter matrix **A** consists of only two unknown parameters *a* and *b* for each benchmark. Parameter matrices **B** and **D** do not exist if do not know the values of the input signal, and structure of design matrix of the first order **C** is well known.

4. EXAMPLE

The case study (Maksimovic, Santrac, 2001) is given as real data example:

A wide area is loaded with 0.1 MN/m^2 *for a short period of time. The terrain profile is a mediaeval plastic clay, with the thickness of* 5.5 m*, horizontally intersected by a sand layer of* 0.5 m *thickness. Underneath the clay layer is an incompressible and impermeable substrate. The clay's module of compressibility is* 5.0 MN/m^2 *. The coefficient of consolidation is* 5*10^{-4} cm^2/s.

On the basis of these data the subsidence is calculated for a period of two years, which is depicted in Figure 4.1. There are 26 epochs (time plan of observations is monthly T=0.0822). Geodetic measures are simulated (height differences) for the assumed subsidence. Adopted accuracy of the height differences is $\sigma = 0.5mm$ (per station). A frame consists of three marks (with labels F1, F2, F3) on a stable terrain. The four marks on the structure all have the same behavior (continual subsidence). The marks and the observation plan are depicted in Figure 4.1.

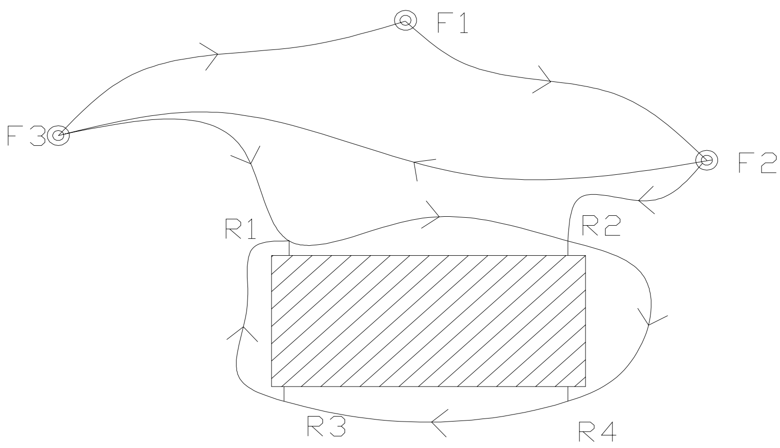


Figure 4.1. Layout of marks with a plan of observations

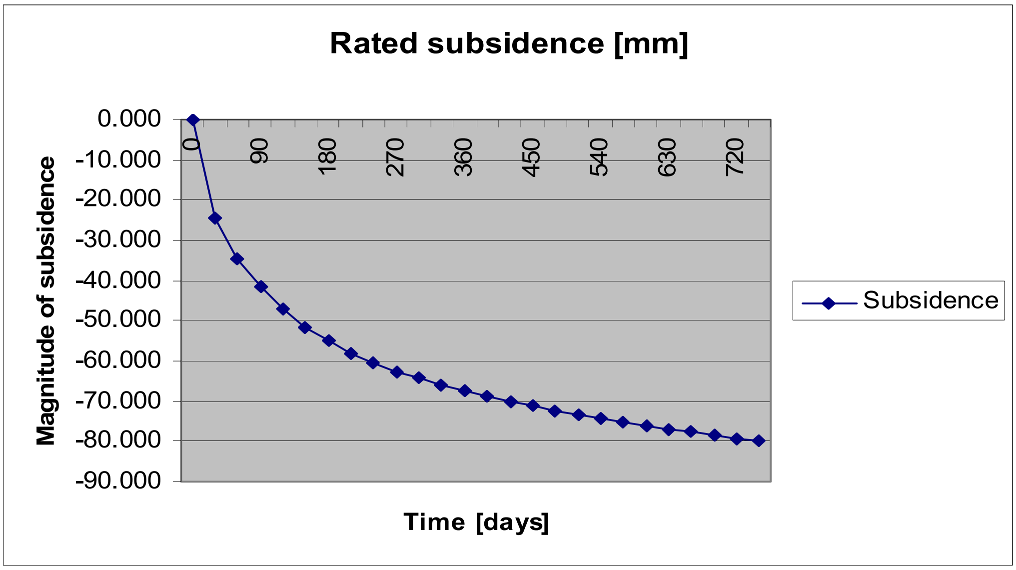


Figure 4.2. Rated subsidence

The rated subsidence of the structure marks depicted in Figure 4.2. Aproximate values of the bench mark are:

Label	H[m]
F1	100.00
F2	101.30
F3	101.20
R1	101.80
R2	101.90
R3	102.00
R4	102.15

The heights of the benchmarks are estimated using the least square method. Because the system is ill-conditioned, LU factorization is applied. On the basis of estimated heights, the first and the second differences are calculated as:

$$\Delta_i^k = \hat{H}_i^{k+1} - \hat{H}_i^k$$
$$\Delta_i^{2^k} = \Delta_i^{k+1} - \Delta_i^k \quad ,$$

and the first derivative, instead of the standard Euler method, is calculated as

$$f_i^{'} = \frac{1}{T}(\Delta_i^k - \frac{1}{2}\Delta_i^{2^k})$$

where k is the number of the epoch.

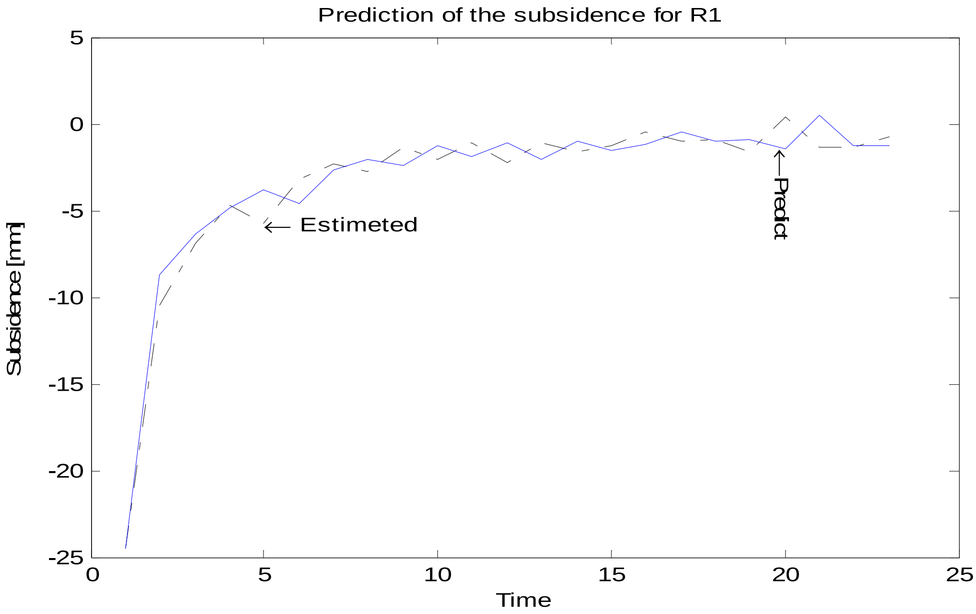
On the basis of five epoch of measurement estimeted parameters a and b. On the next table showed estimated parameters

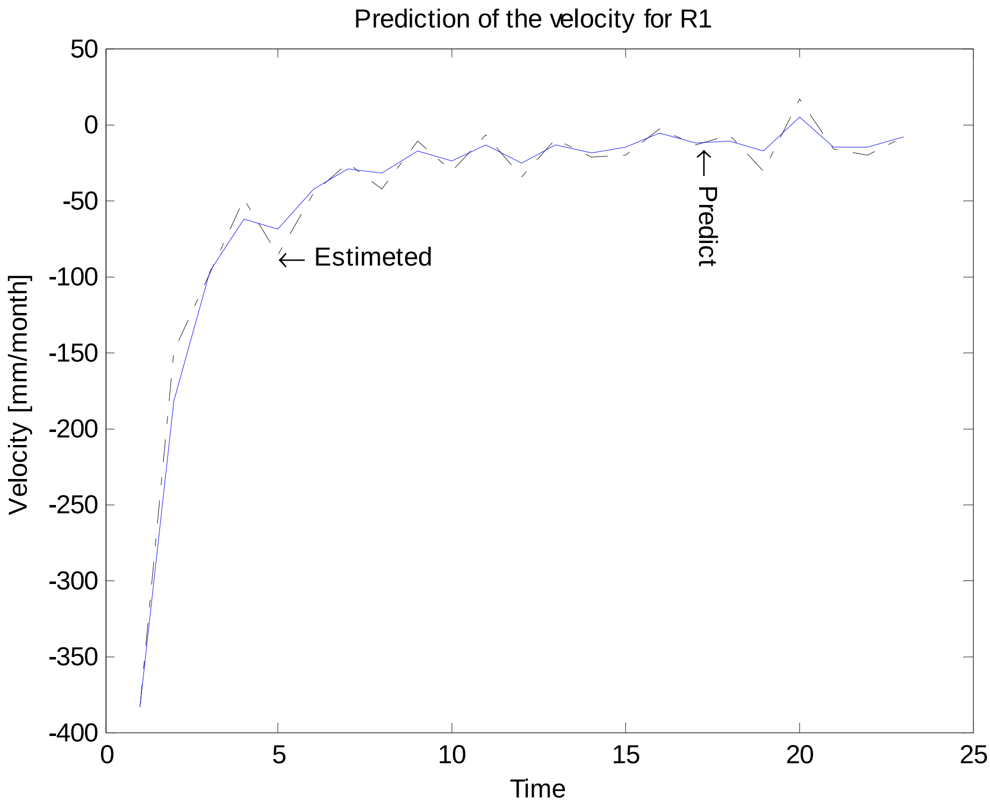
Label	$\hat{a}$	$\hat{b}$
R1	-137.52	6.08
R2	-128.68	5.58
R3	-496.96	34.32
R4	-142.75	6.23

Standard deviations of prediction are shown in the following table.

Label	Subsidence [mm]	Velocity [mm/month]
R1	1.0010	9.9049
R2	1.0489	8.7808
R3	1.2363	60.6044
R4	0.7517	8.9279

Next figures show estimated and predicted subsidence and velocity for the benchmark R1.





5. CONCLUSION

I found the idea for my work at FIG publication NO. 25, in which it is said that engineering surveying may significantly contribute to realistic interpretation of dynamical processes under investigation and that contribution is to be achieved by the combination of techniques developed in the system theory.

The Euler method for the deformation modeling offers geodesists a good alternative for describing structural deformation. The deformation process is non-linear, but we can use successfully this linear method for the subsidence approximation, although the external forces are unknown. Because of the system's ill-conditioning one must use more iterations for the subspace identification procedure.

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BIOGRAPHICAL NOTES

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Study of geodesy at University of Belgrade, Faculty of Civil Engineering, Department of geodesy. Graduated 1992, and take a master degree 1998 at same faculty. Work at Faculty of Civil Engineering, Department of geodesy and geoinformatic since 1992 as a teaching assistant on the subject Engineering Survey. Areas of interes: deformation analisys, identification of system, theory of errors.

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