

Computation of a New Gravimetric Geoid Model over Sweden Using the KTH Method

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Key words: Geoid, GNSS, regional geoid determination, kernel modification

SUMMARY

An important part of the work to construct a new national geoid model for practical surveying in Sweden has been the development of an improved gravimetric model from regional gravity data and a Global Geopotential Model (GGM). It is planned that this model will be modified and adapted for the Swedish reference systems using a number of accurate GNSS/levelling geoid heights, which will result in the new national geoid model. The technique chosen to compute the gravimetric model is the so-called KTH method, developed at the Royal Institute of Technology (KTH) in Stockholm. It consists of a least squares (stochastic) kernel modification with additive corrections for the topography, downward continuation, the atmosphere and the ellipsoidal shape of the Earth.

It is the purpose of this paper to present the computational results of a new gravimetric geoid model over Sweden by the KTH method. This work has been performed in close cooperation between KTH and Lantmäteriet (National Land Survey of Sweden). Traditionally, gravimetric geoid models have been computed over Sweden and the other Nordic countries by the Nordic Geodetic Commission (NKG), the latest model being NKG 2004. Another aim of this paper is to compare the KTH and NKG 2004 models and evaluate their accuracy using GNSS/levelling geoid heights. It is concluded that the new gravimetric model is a significant step forward compared to NKG 2004.

SAMMANFATTNING

En viktig del när det gäller beräkningen av en ny geoidmodell för praktisk användning i Sverige är att förbättra den underliggande gravimetriska geoidmodellen, som beräknas ur regionala tyngdkraftsdata och en global geopotentialmodell (GGM). Den gravimetriska modellen kommer senare att modifieras och anpassas till de svenska referenssystemen med hjälp av ett stort antal noggranna geoidhöjder beräknade ur GNSS och avvägning. Modellen har beräknats i enlighet med den metod som utvecklats vid den Kungl. Tekniska Högskolan (KTH) i Stockholm, vilken brukar kallas för KTH-metoden.

Syftet med denna artikel är att presentera beräkningen av en ny gravimetriska modell över Sverige med KTH-metoden. Arbetet har utförts i nära samarbete mellan KTH och Lantmäteriet. Dessutom jämförs den slutgiltiga modellen med den hittills bästa gravimetriska modell som funnits tillgänglig över Sverige, nämligen NKG 2004. En slutsats är att den nya modellen är ett signifikant steg framåt jämfört med NKG 2004.

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1. INTRODUCTION

National geoid models for practical surveying applications in Sweden are published by Lantmäteriet (National Land Survey). The most recent model is SWEN05_RH2000, which also has a sister model (SWEN05_RH70) adapted to the old national height system; see Ågren and Svensson (2006) and Ågren and Svensson (2007). Up to now, these models have been computed by adapting the latest gravimetric geoid model of the Nordic Geodetic Commission (NKG) to the Swedish reference systems using geoid heights from GNSS/levelling. For instance, the current national model has been derived by modifying the Nordic gravimetric model NKG 2004. The latter model was derived by a remove-compute-restore technique involving topographic corrections by the Residual Terrain Model (RTM) technique and a Wong and Gore type of kernel modification; see Forsberg et al. (2004).

An important part of the work to construct a new national geoid model for practical surveying in Sweden has been the development of an improved gravimetric model from regional gravity data and a Global Geopotential Model (GGM). Later this model will be modified and adapted for the Swedish reference systems using GNSS/levelling, which will result in the new national model. The technique chosen to compute the gravimetric model is the so-called KTH method, developed at the Royal Institute of Technology (KTH) in Stockholm over a long period of time. The method includes least squares (stochastic) kernel modification with additive corrections for the topography, downward continuation, the atmosphere and the ellipsoidal shape of the Earth. The method has been documented in a long row of publications; see for instance Sjöberg (1991, 2000, 2003a, 2003b, 2004 and 2007), Sjöberg and Nahavandchi (2000), Ellmann (2004), Ågren (2004a, 2004b) and Kiamehr (2006a, 2006b).

The main purpose of this paper is to present the computational results of a new gravimetric geoid model over Sweden by means of the KTH method. This work has been performed in cooperation between KTH and Lantmäteriet. Another aim is to evaluate the KTH and NKG 2004 models using a new, very accurate set of GNSS/levelling geoid heights.

Let us end this introduction with a terminological note. Above we have used the term *geoid model* in a general sense, denoting any kind of geoid-like model. However, since normal heights are used in the national height systems of Sweden, it is clear that what we are really talking about here is *quasigeoid models* that describe the variation of the height anomaly. This is also true for NKG 2004, which is a quasigeoid model. Below we will use quasigeoid and geoid in the precise senses of these terms, but resort to talking about geoid models in the general way at the end of the summary. It should be clear from the context what is meant.

2. THE KTH METHOD

Another name of the KTH method that is often used is the *least squares modification method with additive corrections*. Several different versions of this method have been presented during the years; see e.g. Sjöberg (2003b). In this section only the version applied in this paper is treated. We first present the method for estimation of geoid heights properly, which is then transformed for the estimation of height anomalies. It is shown that the latter method is identical to the technique for estimation of height anomalies in Ågren (2004b).

In the least squares modification of Stokes' formula (e.g. Sjöberg 1991), Stokes' kernel is modified in such a way that the expected global mean square error is minimised. This technique can be applied with the standard remove-compute-restore estimator (e.g. Ågren 2004), but according to the KTH practice the so-called *combined estimator* is preferred (Sjöberg 2003b). This means that Stokes' formula (truncated to a cap) is applied to the uncorrected surface gravity anomaly, Δg . After that, the geoid height N is computed by adding a number of additive corrections, i.e.

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \Delta g d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M (s_n + Q_n^M) \Delta g_n^{GGM} + \delta N_{COMB} + \delta N_{DWC} + \delta N_{ATM} + \delta N_{ELL} \quad (1)$$

where σ_0 is the spherical cap, R is the mean Earth radius, γ is mean normal gravity, $S^M(\psi)$ is the modified Stokes' function, s_n are the modification parameters, M is the maximum degree of the Global Geopotential Model (GGM), Q_n^M are the Molodensky truncation coefficients and Δg_n^{GGM} is the Laplace harmonic of the gravity anomaly determined by the GGM of degree n .

The four *additive corrections* to the right in Eq. (1) are derived in such a way that the same result is ideally obtained as when the remove-compute-restore technique is utilised (except for numerical effects). Below we treat them one by one.

The *combined topographic effect* δN_{COMB} is computed as (Sjöberg 2007; Ågren 2004a):

$$\delta N_{COMB} = -2\pi G \rho H_p^2 - \frac{4\pi G \rho H_p^3}{3r_p} \quad (2)$$

where P is the computation point, H_p is the topographic height, $r_p = R + H_p$, G is the gravitational constant and ρ is the density of the topographic masses. This formula has been questioned by Vermeer (2008) as not being exact. He shows, though, that the approximation is good. In any case, since we are estimating height anomalies in the present case, Vermeer's objections are not relevant; see below.

The *downward continuation effect* δN_{DWC} is (Sjöberg 2003a; Ågren 2004),

$$\delta N_{DWC}(P) = \frac{\Delta g(P)}{\gamma} H_p + 3 \frac{\zeta_p^0}{r_p} H_p - \frac{1}{2\gamma} \frac{\partial \Delta g}{\partial r} \Big|_P H_p^2 + \frac{R}{2\pi} \sum_{n=2}^M (s_n + Q_n^M) \left[\left(\frac{R}{r_p} \right)^{n+2} - 1 \right] \Delta g_n^{GGM}(P) \quad (3)$$

$$+ \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \left(\frac{\partial \Delta g}{\partial r} \Big|_Q (H_p - H_Q) \right) d\sigma_Q$$

where ζ_p^0 is an approximate value of the height anomaly and Q is the running point in Stokes' integral.

The *combined atmospheric effect* δN_{ATM} can be approximated to order H by (Sjöberg and Nahavandchi 2000)

$$\delta N_{ATM}(P) = -\frac{2\pi R \rho_A}{\gamma} \sum_{n=2}^M \left(\frac{2}{n-1} - s_n - Q_n^M \right) H_n(P) - \frac{2\pi R \rho_A}{\gamma} \sum_{n=M+1}^{\infty} \left(\frac{2}{n-1} - \frac{n+2}{2n+1} Q_n^M \right) H_n(P) \quad (4)$$

where ρ_A is the atmospheric density at sea level, H_n is the Laplace harmonic of degree n for the topographic height.

The *ellipsoidal correction to the modified Stokes' formula* δN_{ELL} to order e^2 is (Sjöberg 2004):

$$\delta N_{ELL}(P) = \frac{R}{2\gamma} \sum_{n=2}^{\infty} \left(\frac{2}{n-1} - s_n^* - Q_n^M \right) \cdot \left(\frac{a-R}{R} \Delta g_n^{GGM}(P) + \frac{a}{R} (\delta g_e)_n \right) \quad (5)$$

where $s_n^* = s_n$ if $2 \leq n \leq M$ and $s_n^* = 0$ otherwise. Furthermore,

$$(\delta g_e)_n = \frac{e^2}{2a} \sum_{m=-n}^n \{ [3-(n+2)F_{nm}] T_{nm} - (n+1)G_{nm} T_{n-2,m} - (n+7)E_{nm} T_{n+2,m} \} Y_{nm}(P) \quad (6)$$

in which T_{nm} are spherical harmonic coefficients for the disturbing potential. See Sjöberg (2004) for the ellipsoidal coefficients E_{nm} , F_{nm} and G_{nm} .

The estimated geoid height N is converted to the height anomaly ζ by the Sjöberg (1995) relation:

$$\zeta = N - \frac{\Delta g_B(P)}{\gamma} H_P + \frac{1}{2\gamma} \frac{\partial \Delta g}{\partial r} \Big|_P H_P^2 \quad (7)$$

where Δg_B is the simple Bouguer anomaly. If Eqs. (1) to (6) are inserted into Eq. (7), the usual expression for the simple Bouguer anomaly is used and the negligible term to the far right in Eq. (2) is neglected, then we obtain the following combined estimator for height anomaly

$$\zeta = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \Delta g d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M (s_n + Q_n^L) \Delta g_n^{GGM} + \delta\zeta_{COMB} + \delta\zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL} \quad (8)$$

where the combined topographic effect $\delta\zeta_{COMB} = 0$; cf. Sjöberg (2000). The downward continuation effect $\delta\zeta_{DWC}$ becomes

$$\delta\zeta_{DWC}(P) = 3 \frac{\zeta_p^0}{r_p} H_P + \frac{R}{2\pi} \sum_{n=2}^M (s_n + Q_n^M) \left[\left(\frac{R}{r_p} \right)^{n+2} - 1 \right] \Delta g_n^{GGM}(P) + \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \left(\frac{\partial \Delta g}{\partial r} \Big|_Q (H_P - H_Q) \right) d\sigma_Q \quad (9)$$

It should be pointed out that the downward continuation effect in the height anomaly case in Eq. (9) also includes a correction for the fact that the extended Stokes' function is not utilised in Eq. (8); see Ågren (2004b, Subsect. 9.5). Also, at the present level of approximation, the atmospheric and ellipsoidal corrections are the same as in the geoid case, i.e.

$$\delta\zeta_{ATM} = \delta N_{ATM} \quad \text{and} \quad \delta\zeta_{ELL} = \delta N_{ELL}.$$

If Eqs. (8) and (9) are carefully studied, it can be seen that the method is equivalent to analytical continuation to point level using the g_1 term in Moritz (1980, Sect. 45). It differs from Moritz' method in that the least squares modification of Stokes' formula is utilised with improved atmospheric and ellipsoidal corrections.

One problem with using the combined quasigeoid estimator in Eq. (1) or (8) is that Stokes' quadrature is made on the rough surface gravity anomaly, which results in large *discretisation errors*. However, by taking advantage of the remove-compute-restore philosophy for the gridding of a comparatively dense gravity anomaly grid using a smoothing topographic correction, such errors can be counteracted; see Ågren (2004b). This makes it possible to take advantage of the high-frequency information available in the DEM. A practical drawback here is that dense grids are required in rough mountain areas, which can be cumbersome.

Some advantages with the combined estimator are that the “real” importance of the correction terms is apparent and that it is easier to compute the topographic, atmospheric and ellipsoidal corrections in this way. One also avoids the global quadrature strictly required to compute the direct and indirect topographic effects when the remove-compute-restore estimator is used.

3. DATA USED TO COMPUTE GRAVIMETRIC MODEL

In this section the gravity and height data utilised in the computation of the Swedish quasigeoid model are first described. After that, the Global Geopotential Model (GGM) is presented. The section ends with describing the gravity anomaly gridding procedure referred to at the end of the last section.

3.1 Gravity anomaly data

The most important information concerning the surface gravity anomaly data is summarised in the following list:

- 495 614 gravity observations are picked out from the NKG gravity database for the area illustrated in Fig. 1.
- Multiple observations at the same location are cleaned. This step is made by computing the weighted average of all multiple observations at one location. No gross error detection has yet been implemented here.
- Selection of the observation with smallest standard deviation in each compartment of a grid with 2km x 2km resolution using the GRAVSOFTE program SELECT (Forsberg 2003). Gravity anomalies with standard errors larger than 8 mGal are neglected. The same step was carried out for the NKG 2004 model; see Forsberg et al. (2004).

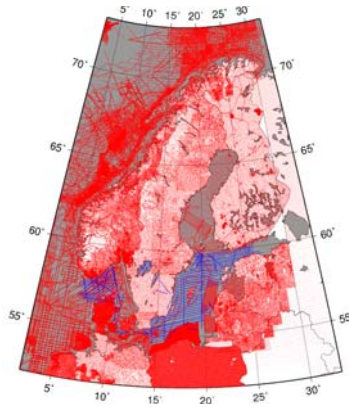


Figure 1: Gravity observations from the NKG database. Blue dots indicate airborne gravity.

3.2 Digital elevation models

Three different DEMs are used in the processing:

- The Nordic elevation model SCANDEM 2004 (Bilker 2004) is first densified to $0.001^\circ \times 0.002^\circ$ resolution (approximately 100 m x 100 m) using the Swedish photogrammetric DEM. In areas not covered by the latter DEM, the denser grid is obtained by interpolation from SCANDEM 2004. The DEM is also extended to the South using SRTM30plus; see http://topex.ucsd.edu/www_html/srtm30_plus.html. This dense DEM is used to compute the Residual Terrain Model (RTM) effect in the remove step of the gravity anomaly gridding procedure; see Sect. 3.3.

- The $0.001^\circ \times 0.002^\circ$ DEM is averaged to $0.01^\circ \times 0.02^\circ$ resolution. These heights are used in the restore step of the gridding and are also utilised to compute the downward continuation effect in Eq. (9). This grid coincides with the gravity anomaly grid.

- Spherical harmonic coefficients for the global topography are needed to compute the combined atmospheric correction by Eq. (4). A worldwide $15' \times 15'$ DEM is first derived from SRTM30plus, updated with the above DEM over the Nordic area. Spherical harmonic coefficients are then estimated to the maximum degree 720 using numerical integration according to the midpoint rule.

3.3 Choice of Global Geopotential Model (GGM)

The GGM used in the computation of the new Swedish quasigeoid model was chosen by evaluating a number of different GGMs with respect to GNSS/levelling. Most of the tested GGMs have been derived in one way or the other from the GRACE mission. The tests were made both by testing the GGMs by themselves and by applying all steps of the KTH-method; see Ågren et al. (2006). The GGM that obtained the best result is constructed using GGM02C (Tapley et al. 2005) up to degree 200. Above that, up to degree 360, the EGM 96 coefficients (Lemoine et al. 1998) are utilised. The model is similar to, but not identical with, the GGM used to compute the Nordic model NKG 2004.

The chosen GGM is of the combined type, which means that the high-frequency information stems from more or less the same gravity anomalies as are used in the least squares

modification of Stokes' formula. It might be argued that a satellite-only model should be preferred instead. One problem with a combined model is that correlations arise between the errors in the GGM and the terrestrial gravity anomalies, which are not considered in the present version of the least squares modification technique; see Sjöberg (1991) and Ågren (2004). However, since a rather pragmatic method is used to choose the least squares modification coefficients (see Sect. 4), which gives priority to the terrestrial data for the high frequencies, it is believed that this problem is of a rather academic nature. In any case, the GGM that yielded the best fit to GNSS/levelling is GGM02C/EGM 96 with $M = 360$, which is the reason for preferring this model and maximum degree for the final model.

3.4 Gridding of the gravity anomalies

As mentioned above, a remove-compute-restore strategy is utilised to reduce the discretisation errors. The resolution of the grid is chosen to $0.01^\circ \times 0.02^\circ$. Denser resolutions have also been tested, but it has been found that the final result is practically the same for the whole country of Sweden. The following gravity anomaly effects are reduced and restored after gridding:

- The long-wavelength effect from the chosen GGM (GGM02C/EGM96) with the maximum degree $M = 360$.

- The high-frequency part of the topographic effect computed by the RTM method with a smooth reference surface (corresponding to M). The TC and TCFOUR programs in GRAVSOF (Forsberg 2003) are applied for this task. The densest $0.001^\circ \times 0.002^\circ$ DEM is used in the remove step, while the $0.01^\circ \times 0.02^\circ$ DEM is chosen in the restore step. This amounts to some smoothing of the topography. It should be mentioned that the indirect geoid effect of this smoothing has been checked to be negligible everywhere in Sweden.

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The gridding of the reduced gravity anomalies is made in two steps:

- Search for gross errors by cross validation. Each reduced observation is predicted from its neighbours using inverse distance interpolation. If the obtained difference is larger than 20 mGal, then the observation is rejected. It might be argued that this rejection limit is too low. However, since comparatively few observations are neglected in Sweden, the limit seems suitable for the present project. At this point, no detailed investigation has been made of the observations with large cross validation residuals.

- The gridding is made by least squares collocation using individual weights for the reduced gravity anomaly observations. The GRAVSOF program GEOGRID (Forsberg 2003) is utilised with 15 km correlation length for the 2nd order Markov covariance model chosen empirically. It should be mentioned that several different interpolation methods have been evaluated, for instance the inverse distance method (Bjerhammar's deterministic method, Bjerhammar 1973, p. 324), but the best results are obtained by least squares collocation with individual weights as implemented in the GEOGRID program.

4. PRACTICAL APPLICATION OF THE KTH METHOD

It is the purpose of this section to describe how the final quasigeoid model was computed from the practical point of view. A number parameters need to be chosen to obtain a good

result. In this section we first discuss the choice of parameters in the least squares modification method. We then take a look at the computation of the additive corrections for the final model.

4.1 Choice of integration cap and degree variances

An important parameter is the spherical radius of the integration cap σ_0 in Eq. (8). A cap with radius $\psi_0 = 3^\circ$ has earlier been found to yield the best fit to Swedish GNSS/levelling (Ågren et al. 2006), which motivates that this radius is used also in the present computation. It is more difficult to choose the parameters of the least squares modification method. It is important to apply realistic weights to the GGM and to the terrestrial gravity anomalies, which is made by specifying error degree variances for these two components. Besides this, signal degree variances also need to be specified. The least squares modification then singles out the modification with the lowest expected global mean square error (Sjöberg 1991). In reality, the modification is naturally only best in this sense in case the assumptions are fulfilled. The standard objection against least squares (stochastic) kernel modification is that we do not know the errors of terrestrial gravity well enough, which is certainly true to some extent, but even so a distinct criterion to match various error sources (like least squares theory with rather crude weighting) should be preferred. In this work a rather pragmatic method is used to choose the error degree variances; cf. Ågren et al. (2006). Below this method is briefly explained.

It should first be noticed that the error degree variances of the GGM tend to increase with the degree. This is at least the case for the frequency band in which the GGM has been determined using dynamic satellite observations; cf. Fig. 2 below. It seems natural to assume that things are the other way around with the terrestrial gravity anomalies, i.e. that the gravity anomaly degree variances decrease with the frequency. This depends on datum problems, systematic errors and like effects, well-known to anyone working with terrestrial gravity data.

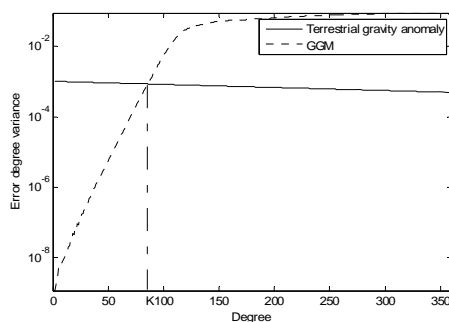


Figure 2: Illustration of the spherical harmonic degree K for which the error degree variances for the GGM and terrestrial gravity are equal.

In the present method, the published error degree variances are used for the GGM. Possibly they are rescaled to become more realistic for the region in question. The signal degree variances are generated according to the Tscherning and Rapp (1974) model. If needed, they are also rescaled. A model for the error degree variances of the terrestrial gravity anomalies is

then assumed that is a combination of white noise and the reciprocal distance model (Moritz 1980, Sect. 23; see also Ågren 2004). With this model the error degree variances decrease with the degree (for low and medium degrees). For very high frequencies, the white part of the noise makes the degree variances increase again, but this fact is not important for the choice of modification parameters. Using the above assumptions, the error degree variances of the GGM and terrestrial gravity look as in Fig. 2, where the two curves are crossing each other at a certain point. The degree at which this crossing occurs is denoted by K ; see the illustration in Fig. 2.

The question now is how to specify the numerical parameters for the gravity anomaly error model (standard errors, correlation length, etc.). In the present method little attention is paid to these parameters. Instead, the focus is on the crossing degree K . It has been found empirically that the choice of K is crucial for the behaviour of the least squares modification. Different parameter choices with the same crossing K tend to yield very similar modifications (but not similar propagated RMS values).

The choice of K amounts to specifying the upper degree for which the GRACE model is believed to be better than (or as good as) the terrestrial gravity anomalies. This corresponds to the specification of an abrupt degree in the Wong and Gore modification. One advantage of the least squares modification is that the parameters are chosen so that the truncation error is low, also for comparatively small caps. This is not the case for the Wong and Gore method, which usually requires a larger integration area; see Ågren (2004) and Sjöberg (2005).

Assuming the above method, Ågren et al. (2006) found that a weighting scheme with $K = 85$ yielded optimal fit to GNSS/levelling with the same GGM, which seems fairly realistic. Based on these tests, $K = 85$ and the cap size $\psi_0 = 3^\circ$ was used to compute the final gravimetric model

4.2 The additive corrections

The purpose of this section is to study the additive corrections for the final quasigeoid model. As mentioned in Sect. 2, one advantage of the present method is that the “real” magnitude of the corrections becomes apparent, i.e. the sum of the additive corrections are equal to the quasigeoid errors obtained in case no corrections are applied at all.

The *downward continuation correction* is computed according to Eq. (9) using the chosen combined GGM (GGM02C/EGM96) with $M = 360$. The downward continuation to point level (Moritz 1980; Sect. 45) is approximated using the vertical gradient of the gravity anomaly. This quantity is calculated by the standard formula Eq. (2-217) in Heiskanen and Moritz (1967) using the surface gravity anomaly grid; see Sect. 3.4. The magnitude of the downward continuation correction is illustrated to the left in Fig. 3.

It is clear that the downward continuation correction is large in, and close to, the mountainous areas in the Northern and Western parts of Sweden. In the lowlands, however, the correction is small; at the 1-cm level. Another observation that can be made in Fig. 3 is

that the high-frequency details are small, usually below 1 cm. It is concluded that it is important to take this correction into account. It should be pointed out that the correction has been computed using a first-order approximation; cf. Moritz (1980, Sect. 45). It is believed that this approximation is sufficient in the region. In any case, as is shown by the synthetic model investigations in Ågren (2004), it is extremely difficult to perform the downward continuation more strictly in practice, using for instance inversion of Poisson's integral.

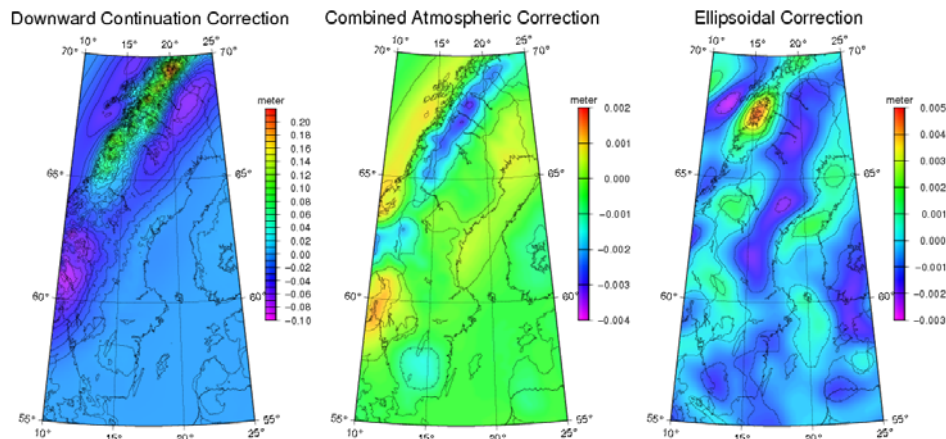


Figure 3: The three additive corrections. Unit: m.

The combined atmospheric correction is computed using Eq. (4) from the spherical harmonic expansion of the global topographic elevation truncated to the maximum degree 720; see Sect. 3.2. The maximum degree of the GGM is still $M = 360$. The correction is illustrated in the middle plot of Fig. 3. The most important thing to notice is that the combined atmospheric correction is small in the present case.

The ellipsoidal correction to the modified Stokes' formula is computed according to Eqs. (5) and (6) using the selected GGM02C/EGM 96 model with maximum degree $M = 360$. The result is given to the right in Fig. 3. The ellipsoidal correction is a little bit larger compared to the atmospheric counterpart in Fig. 3, but it is still small, well below the 1 cm level.

Both the atmospheric and ellipsoidal corrections are thus small for the chosen version and parameters of the least squares modification method. It should be noted, though, that the above additive corrections depend on the modification method, cap size and maximum degree of the GGM being used; cf. Ellmann (2004). Consequently, one should not jump to conclusions concerning their *general* importance based on the *particular* results presented in this section. It is good practice to check the magnitude of the atmospheric and ellipsoidal corrections for the modification method, cap size and maximum degree of the GGM that one uses.

5. EVALUATION USING GNSS/LEVELLING DATA

The new gravimetric quasigeoid model for Sweden was computed using the above theory, data and parameter choices. In what follows it is referred to as the KTH model. In this section the model is evaluated using GNSS/levelling observations and a comparison is made with the best quasigeoid model previously available, namely the Nordic NKG 2004 model.

To evaluate the gravimetric models 195 high-quality GNSS/levelling height anomalies in the reference systems SWEREF 99 and RH 2000 are available; the locations are illustrated on the maps of Fig. 4. The normal heights in RH 2000 either have been determined in the RH 2000 adjustment or by utilising high quality levelling connections. The coordinates of the SWEPOS stations are very well determined. In fact, 21 of them define SWEREF 99. All other GNSS heights have been derived using at least 48 hours of observations, Dorne Margolin T antennas and processing in the Bernese software. Below the latter are referred to as SWEREF stations. Statistics on the GNSS/levelling observations and approximate standard errors are summarised in Tab. 1.

Table 1: The GNSS/Levelling observations and their approximate standard errors.

Data set	#	Short description	Appr. standard errors (mm)		
			GNSS height	Normal height	Height anomaly
SWEPOS	24	Permanent GNSS stations whose coordinates define SWEREF 99	5-10	5-10	7-14
SWEREF	171	Determined relative to SWEPOS using 48 hours of obs, DM T antennas and the Bernese software	10-20	5-10	11-22

The GNSS/levelling residuals after a 1-parameter transformation (fit) of the KTH and NKG models are plotted in Fig. 4. The statistics after both a 1- and a 4-parameter transformation are given in Tab. 2.

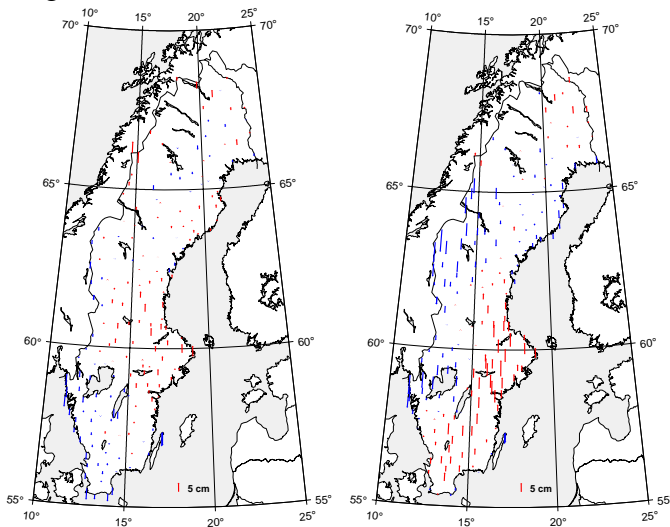


Figure 4: GNSS/levelling residuals for the KTH model (left) and for the NKG 2004 model (right) after a 1-parameter transformation. The scale is given by the 5 cm arrow to the South-East.

Let us first discuss the fit of the KTH model. As can be seen in the left plot of Fig. 4, the agreement is good and homogeneous for the whole of Sweden, even though there are some large residuals along the borders. Another problem is that there seem to be a systematic bulge in the central part of the country. However, if the estimated standard errors of the GNSS/levelling (Tab. 1) are considered, then the main conclusion is that the resulting height anomalies are very good. For instance, if the standard errors for GNSS and levelling are

taken as 15 mm and 7.5 mm, respectively (middle of the intervals in Tab. 1), then the propagated standard error for the 4-parameter fitted gravimetric quasigeoid becomes 11 mm.

Table 2: Comparison of the KTH and NKG 2004 models with GNSS/levelling. Statistics for the 195 GNSS/levelling residuals after a 1 and a 4 parameter fit. Unit: mm.

Geoid model	# par	Min	Max	Mean	StdErr
KTH	1	-74	64	0	23
	4	-73	70	0	20
NKG 2004	1	-125	89	0	40
	4	-106	77	0	32

Next, the KTH model is compared to the NKG 2004 counterpart. As discussed in the introduction (Sect. 1), NKG 2004 was computed by the remove-compute-restore method using the RTM reduction and a Wong and Gore type of modification, selected to high pass filter the gravity anomalies above the degree 30. The NKG 2004 model is documented in Forsberg et al. (2004). It should be pointed out, however, that this documentation is of a preliminary version of NKG 2004. The final model was computed in almost the same way, the largest difference being that the then brand new GRACE model GGM02S (Tapley et al. 2005) was applied.

It should be pointed out that the NKG 2004 and KTH models have not been computed using exactly the same data. Of course, this implies that the comparison cannot be absolutely conclusive with respect to the computation method. However, it is still the case that almost the same observations are applied. Approximately the same GGM are utilised in both cases (GGM02C/S and EGM 96 for the highest degrees to $M = 360$). The same gravity data from the NKG database are also used (at least for the overlapping area). The difference here is that the data have been handled in different ways. The applied DEMs also differ somewhat, since a denser DEM is used over Sweden for the KTH model; see Sect. 3. However, in Ågren et al. (2007) the method used for NKG 2004 was applied on exactly the same data as used for the KTH model. The results clearly show that the differences from using not exactly the same data are very small in the present case.

It seems clear that the NKG 2004 model is significantly worse compared to the KTH model. Since the residuals are so small in the KTH case, it seems almost certain that the main parts of the NKG 2004 errors are located in the quasigeoid model and not in GNSS and/or levelling. It can be further seen that the main difference between the models is that the long-wavelength errors are larger for NKG 2004. Most likely, they are caused by various systematic errors among the gravity anomalies. Since the KTH model makes use of the GRACE model up to degree 85, which is significantly higher than for NKG 2004 modification (degree 30), it becomes possible to high-pass filter the gravity anomalies more efficiently. However, the computation methods differ in many ways, which makes it difficult to say exactly which differences that cause the largest improvements.

6. SUMMARY

The main purpose of this paper has been to present the computation of a new gravimetric geoid model over Sweden and to compare this model to the best model so far available in the area, i.e. NKG 2004. The work is a joint undertaking between KTH and Lantmäteriet.

After presenting the applied version of the KTH method in Sect. 2, the data used has been described in Sect. 3. In the next section some important processing options have been discussed, namely the choice of integration cap radius, gravity anomaly weighting and the computation of the additive corrections. In Sect. 5, the computed quasigeoid model (the KTH model) has first been carefully evaluated. The 20 mm standard error obtained by the KTH-model in the 4-parameter fit is very satisfying. If the standard errors for GNSS and levelling are taken as 15 mm and 7.5 mm, respectively (which is reasonably realistic), then the propagated standard error for the fitted gravimetric quasigeoid becomes as low as 11 mm. After that, the NKG 2004 model has been evaluated in the same way. The latter results are significantly worse compared to the KTH case. It is believed that the main improvement is due to the reduction of the long-wavelength errors; cf. for instance the systematic bulges south of the 60° latitude.

It is planned that the new gravimetric quasigeoid model (KTH model) will be used by Lantmäteriet to compute a new national geoid model for Sweden. The gravimetric model will then be modified and adapted to the Swedish reference systems using GNSS/levelling.

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BIOGRAPHICAL NOTES

Dr. Jonas Ågren, born in 1967. Graduated in 1994 from the Royal Institute of Technology (KTH) in Stockholm as a land surveyor with emphasis on Geodesy and Photogrammetry. He then worked with GPS and reference systems at Lantmäteriet until 1998. He then continued his studies and obtained his doctoral degree from KTH in 2004. Since then he is working with geoid determination, absolute Gravimetry and geodynamics at Lantmäteriet.

Dr. Ramin Kiamehr, born in 1968 in Urmia city of Iran. He completed has BSc and MSc degrees from K.N. Toosi university of Technology in Tehran. Then he continued his studies and obtained his doctoral degree from KTH in 2006. Since December 2006 he is working in the position of assistant professor in department of Geodesy and Geomatics of Zanjan University in Iran. His main interest of research areas are geoid computation, geodynamics and their geophysical interpretations and applications in engineering. Ramin Kiamehr selected as best lecturer and researcher of Zanjan University in 2003. He has publication of a book titled “Engineering Surveying” and a chapter of books titled “Dynamics Planets” and publication/presentation of more than 40 articles in his CV.

Prof. Lars E. Sjöberg, born in 1947, graduated as land surveyor in Surveying and Mapping at KTH in 1971, and he got his Ph.D. in Geodesy at the same institute in 1975. After post-doc positions at KTH, the Ohio State University and the NLS at Gävle, he returned to KTH as the

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