Development of the Geoid-Ellipsoid Separations Model

$H = h - N$

H = orthometric height
h = ellipsoidal height
N = ellipsoid-geoid separation

Objectives :
to determine the values of orthometric heights by means of the GPS measurements.

DATA USED FOR THE DEVELOPMENT OF THE MODEL :

- **GPS stations network**
- **State leveling network**

Development of the Geoid-Ellipsoid Separations Model in Israel

Dr. Yakov Tuchin
Survey of Israel

Development of the GPS network in Israel 1994 - 2001

Orthometric Heights Network

Count of Benchmarks 1 – 3 classes : 5081

Count of Benchmarks that have ellipsoidal heights : 684

Development of the GPS network in Israel 1994 - 2001

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Count of points in the net : 895

Development of the GPS network in Israel 1994 - 2001

Construction of the Geoid-Ellipsoid Surface

The surface constructed on the basis of 684 stations

Using Kriging interpolation (optimal prediction) method

$U_n = U(x_n, y_n)$ – value of separation in the point (x_n, y_n)
Is calculated as weighted linear combination of neighboring values of U_i

$$U_0 = \sum_{i=1}^{n} W_i * U_i$$

$$\sum_{i=1}^{n} W_i = 1$$

The error is:

$$r_n = U_n - \sum_{i=1}^{n} W_i * U_i$$

Ellipsoidal Heights Network Adjustment

Count of points in the net : 895

Count of vectors :2343
r.m.s :0.023
Deviation :0.018

Stations with errors exceeding r.m.s

Count of Fixed points : 101

Error exceeding :
0.050 m - 21 stations
(between 0.050 and 0.060 m)

0.060 m - 13 stations

Variogram

The ordinary Kriging method implies that, instead of covariance for the weights calculations, variogram is used.

$$g(\Delta x, \Delta y) = \frac{1}{2} e[\{U(x+\Delta x, y+\Delta y) - U(x,y)\}^2]$$

$e[]$ - statistical expectation operator

$$s^2_{U_0} = \sum W_i g(U_i, U_0) + m - g(U_0, U_0)$$

and the error variance is:

$$s^2_{R_0} = s^2 + \sum\sum W_i W_j \operatorname{cov}(U_i, U_j) - 2\sum W_i \operatorname{cov}(U_i, U_0) + 2m(\sum W_i - 1)$$

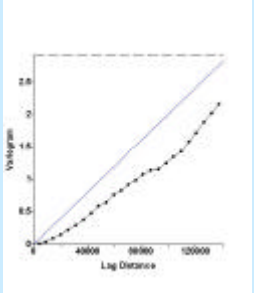
$$s^2 = \operatorname{cov}(U_0, U_0)$$

m is the Lagrange parameter

Weights are defined under the condition
of the minimum error variance which leads to the system of equations:

$$\sum W_j \operatorname{cov}(U_i, U_j) + m = \operatorname{cov}(U_i, U_0)$$

Variogram for resolution 4 x 4 km



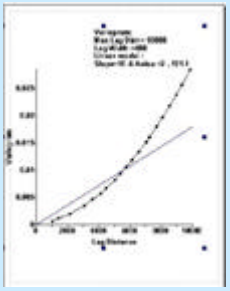
Some technical details

Separations grid was built in 3 steps:
the first resolution being 4 x 4 km,
the second 1 x 1 km
the final 0.5 x 0.5 km.

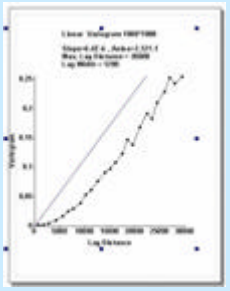
Scattered data interpolation for all the steps was:

search radius (km)	218
number of sectors to search	4
maximum number of data to use from each sector	6
minimum number of data in all sectors	5

Variogram
for resolution 0.5 x 0.5 km

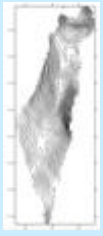


Variogram
for resolution 1 x 1 km

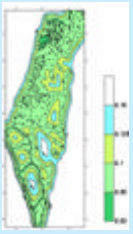


Realization of the Model

The 684 points of the model



The isolines of the geoid-ellipsoid separations

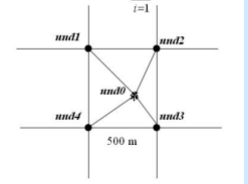


The isolines of the model accuracy values

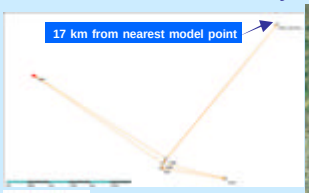
Realization of the Model

Undulations calculation using 0.5 x 0.5 km resolution grid

$$und_0 = \sum_{i=1}^{i=4} und_i * 1/r_i^3 \Big/ \sum_{i=1}^{i=4} 1/r_i^3$$



Model validation by measurements



The orthometric height of the station 543U is 533.442

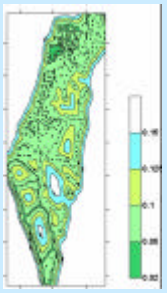
Realization of the Model

Separation values vary from 16 to 24 m

Separation accuracy varies from 3.5 cm to 18 cm

The highest accuracy is achieved in the central and northern parts of country

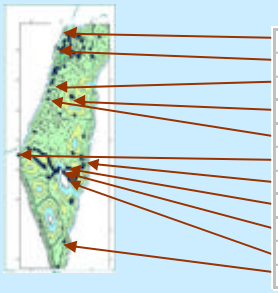
The least reliable separations are achieved in the deserts of Negev and Arava and the area of the Dead Sea



Model validation by measurements

Name	MODEL		H_Datum98	Datum98 - model
	Accuracy	H_orthometric		
774U	0.053	7.972	7.991	0.019
BSHM	0.046	204.580	204.614	0.034
794U	0.037	23.855	23.809	-0.046
ARIL	0.042	633.883	633.910	0.027
TELA	0.067	39.711	39.785	0.074
940W	0.066	94.949	95.002	0.053
3357MPI	0.051	-353.581	-353.598	-0.017
6211	0.133	385.847	385.725	-0.122
YR3	0.137	484.327	484.238	-0.089
602U	0.154	363.878	363.847	-0.031
NRIF	0.034	440.054	440.061	0.007

Model validation by measurements



Thank you !