A PRECISE GEOIDAL MAP OF THE SOUTHERN PART OF EGYPT, BY COLLOCATION: TOSHKA GEOID

Maher Mohamed AMIN (Egypt) **Saadia Mahmoud EL-FATAIRY(Egypt)**
Raaed Mohamed HASSOUNA (Egypt)

Key words: geoid, collocation, tailored models, data validation

ABSTRACT:

This paper presents the final results of computing a 5'x5' grid values of geoidal heights, of the southern part of Egypt that lies between (22°N≤φ≤25°N; 28°E≤λ≤34°E), where the government have begun one of the mega national projects of developmental work, in virgin areas of Egypt's vast deserts, named Toshka project. The ultimate goal was to render a high precise geoid map that serves the newly geodetic activities undertaken in the considered area. In the current investigation, we have exploited all the old and the recent available heterogeneous geodetic data in the same area to compute the required values of geoidal heights by using the Least-Squares Collocation (LSC) technique. The remove-restore procedure was adopted, by the aid of an appropriate high degree global harmonic model, tailored to the local terrestrial data. A digital terrain model for Egypt was also used for the same purpose. The final results of comparing the predicted geoidal heights with the observed ones at certain check points, have shown a rather good external accuracy that reaches 16 cm.

1 INTRODUCTION

Recently, the virgin desert area of the southern part of Egypt have seen the beginning of a revolutionary era of agricultural and industrial development, since the Egyptian government have decided to begin one of its national mega projects for development in that area, named Toshka project. This led to the need for accurate maps of different kinds for this long time neglected area. The accuracy required for such maps necessitates, in turn, the knowledge of the accurate and precise geoid at the same area. This is why we have decided to make this study, especially after the relevant data of that area have been available to us. It should be noted that the region under study may be considered as composed of two areas, which were separately investigated by (Abd-Elmotaal,1998 and Tscherning et. al., 2001). However, in the current study, a locally fitted geopotential model is used as a reference field, besides, a DEM is utilized for smoothing the available data. After the (LSC) prediction, the effects of both removed parts were added to the predicted signals, at the computation points each 5'x5', to get the final geoid values, relative to the WGS-84 ellipsoid. The operation of subtracting and adding the effect of the systematic parts of the data, before and after the prediction process, is usually known in literatures as the Remove-Restore technique The final results were then used to produce two contour maps, representing the geoid and its error standard deviations, for the whole studded area.

2 DATA TYPES, REDUCTION, FILTRATION AND SMOOTHING

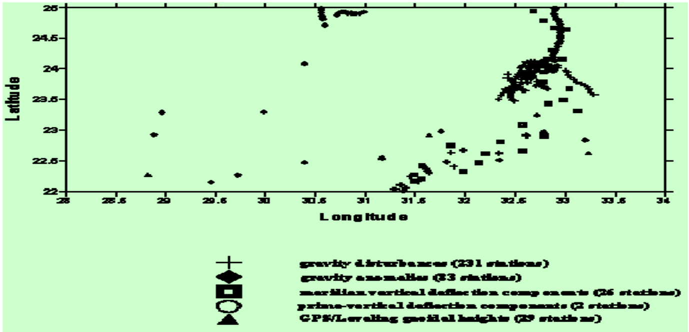

Figure (1): Heterogeneous data distribution in the considered area

Long and short wavelength contributions

In the current investigation, we have used a tailored model, denoted as EGM96EGCT, generated by the authors, using the LSC method, and estimated up to degree and order 599, based on the coefficients of the global model EGM96 and all the available terrestrial geodetic data in Egypt. The reliability of this tailored model to recover the long wavelength features of the Egyptian gravity field in an efficient manner has been verified (Amin et al., 2003b). In a previous work by the authors (Amin et. al, 2003a), a detailed 5'x5' digital elevation model DEM for Egypt was computed by collocation, based on the available local height data and the global high-resolution topographic harmonic model GTM3A. Using the RTM method, this DEM model has been utilized in the current study for removing the high frequency features.

Table (1): Statistics of original and residual gravity anomaly data (unit: mgals)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity anomaly	4.965	17.857	18.431	-25.688	69.005
RTM reduced gravity anomaly	11.244	19.136	22.095	-20.911	75.002
Final (RTM + EGM96EGCT) residual gravity anomaly	4.092	16.591	16.991	-24.632	51.277

Table (2): Statistics of original and residual gravity disturbance data (unit: mgals)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Free air gravity disturbance	-2.159	8.182	8.445	-23.389	18.032
RTM reduced gravity disturbance	0.597	8.434	8.437	-19.081	22.584
Final (RTM + EGM96EGCT) residual gravity disturbance	-0.815	6.775	6.810	-19.021	17.858

Table (3): Statistics of original and residual meridian deflection component data (ξ) (unit: arc-seconds)

Item	Mean	Std. Dev.	RMS	Min.	Max.
ξ	-0.789	2.351	2.437	-7.021	3.380
RTM reduced ξ	-0.924	2.206	2.353	-6.637	2.588
Final (RTM + EGM96EGCT) residual ξ	-0.243	1.785	1.767	-3.660	3.346

Table (4): Statistics of original and residual prime-vertical deflection component data (η) (unit: arc-seconds)

Item	Mean	Std. Dev.	RMS	Min.	Max.
H	-5.527	1.527	5.632	-6.607	4.448
RTM reduced η	-5.465	1.575	5.577	-6.578	-4.351
Final (RTM + EGM96EGCT) residual η	-1.585	1.656	1.971	-2.756	-0.414

3 COMPUTATION TECHNIQUE, PROCEDURES AND RESULTS

Local covariance function

Since the dominant data type used in this particular work was the gravity disturbance, the computed residual parts δg_r of this element were therefore depicted and utilized to determine this function. Practically, this have been done by evaluating the product sum average of pairs of gravity disturbance signals, relevant to point pairs having spacing $(\psi-\Delta\psi)/2 \leq \psi \leq (\psi+\Delta\psi)/2$.

In order to perform the different steps of calculation by collocation, we must have a covariance function model that represents the local area well. This is usually achieved by fitting the estimated values of the empirical covariance function to a model function in a none-linear iterative least-squares adjustment with three parameters. In this respect, we have used the well-known formula of the anomalous potential covariance function model that reads (Tscherning, 1993),

$$C(P,Q)= C(r , r', \psi)$$

$$= \sum_{n=2}^{N\max} c.\sigma^2_{neT.model} \left(\frac{R_b^2}{rr'}\right)^{n+1} P_n(\cos\psi) \;+$$

$$\left(\frac{R_b^2}{rr'}\right)^{n+1} \qquad (1)$$

with

Nmax = maximal degree of the removed reference field (= 599)
C = 0.001037
A = 15.627 mgal^2
R_B - R = -249.682 m

Figure (2) illustrates the input residual gravity disturbance isotropic empirical covariance function, and its associated fitted analytical function. The obtained final values were then used as input for the collocation process.

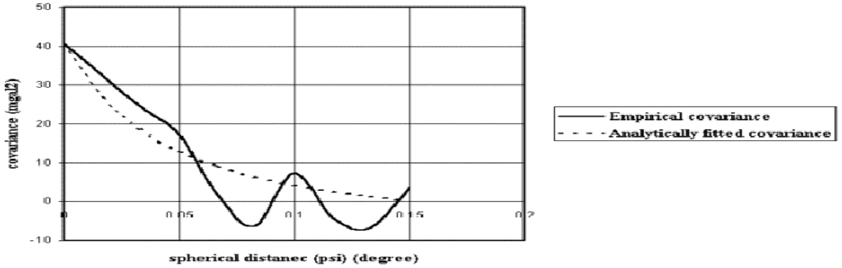

Figure (2): Residual gravity disturbance empirical and fitted covariance functions

Least-squares collocation procedures and computations

The vector of the residual geoid undulation N_r is computed from the vector of observations x from which the GGM and DTM contributions have been computed and removed ,

$$N_r = C_{Nx} \left(C_{xx} + C_{nn} \right)^{-1} x_2, \qquad (2)$$

where C_{xx} and C_{Nx} are the auto- and cross- covariance matrices between the observations and the predicted geoid, as evaluated from the computed parameters of the selected analytical model; and C_{nn} is the error (noise) covariance matrix of the observations. The error covariance matrix of the estimated geoid undulations is given by

$$C_{ee} = C_{NN} - C_{Nx} \left(C_{xx} + C_{nn} \right)^{-1} C_{xN}, \qquad (3)$$

where C_{NN} is the auto-covariance matrix of N.

4 FINAL RESULTS, CONCLUSIONS AND RECOMMENDATIONS

The contributions of both the tailored harmonic model and DEM were then added back (restored) to the residual geoid values predicted at the 5'x5' grid nodes, in order to obtain the respective 5'x5' full spectrum geoid values as well as their error estimates. Table (5) shows the statistics of the predicted geoid. Table (6) shows the accuracy, resulting from the comparison at GPS/Lev. check points, which have not been used as data points. Finally, Figures (3) and (4) show the contour maps of the geoid and its error standard deviations, respectively.

Table (5): Statistics of the residual and full predicted 5'x5' Toshka geoid
(Unit: meters)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Residual N	0.038	0.059	0.071	-0.074	0.390
Final N	11.036	1.819	11.185	7.489	16.641
Std. error	0.046	0.003	0.046	0.019	0.047

Table (6): Statistics of the differences among the observed and predicted geoidal heights
at GPS/Leveling check stations (Unit: meter)

Item	Mean	Std. Dev.	RMS	Min.	Max.
N(obs.) – N(pred.)	0.017	0.161	0.159	-0.355	0.380

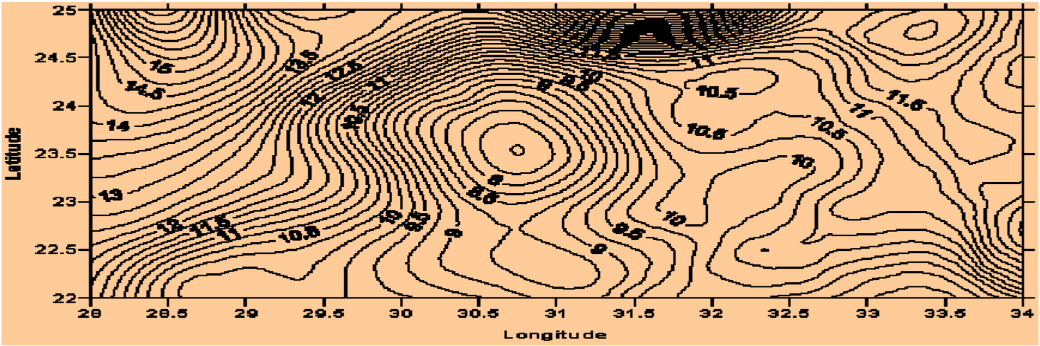

Figure (3): Contour map for the 5'x5' Toshka geoid (Interval: 0.25 m)

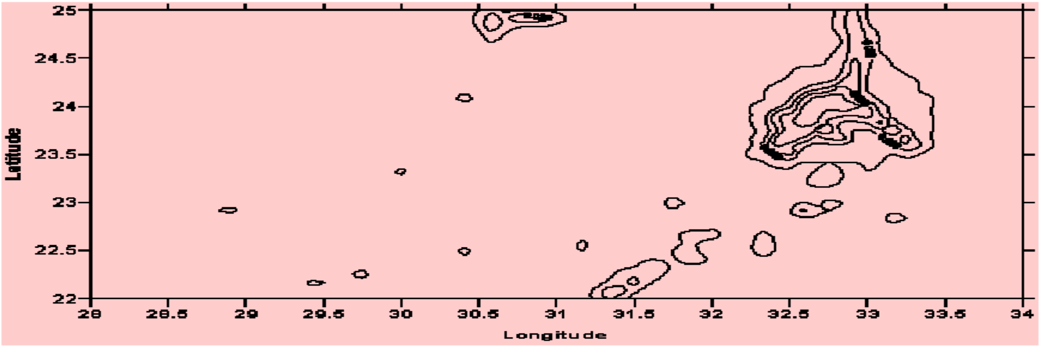

Figure (4): Contour map for the standard errors of the 5'x5' Toshka geoid (Interval: 5 mm)

Therefore, based on the results, the obtained Toshka geoid is highly recommended to be used for any future geodetic computation in the newly developed sector of the southern part of Egypt. It is also recommended to be utilized for densifying leveling networks of lower order through GPS observations, especially in remote areas of the western or eastern desert of that part of Egypt. Consequently, appropriate future planning for rigorous GPS measurements in these areas is highly recommended.

REFERENCES

Abd-Elmotaal, H. (1998): "Detailed Gravimetric Geoid for the Egyptian South-Western Desert", Phys. Chem. Earth, Vol. 23, No.1 :77-80.

Abd-Elmotaal, H. and El-Tokhey, M. (1997): "The Role of the Geoid in the Coordinate Transformation", Survey Review, 34, 263, January: 31-40.

Amin, M.M. (2002): "Evaluation of Some Recent High Degree Geopotential Harmonic Models in Egypt", Port-Said Engineering Research Journal PSERJ, Published by Faculty of Engineering, Suez Canal University, Port-Said, Egypt, Vol. 6, No. 2, September: 442-458.

Amin, M.M.; El-Fatairy, S.M. and Hassouna, R.M. (2003a): "A Digital Elevation Model for Egypt by Collocation", Scientific Bulletin of Matarya Faculty of Engineering, Helwan University, Cairo, Egypt, , Vol. 85, February: 110-127.

Amin, M.M.; El-Fatairy, S.M. and Hassouna, R.M. (2003b): "Two techniques of tailoring a global harmonic model: operational versus model approach applied to the Egyptian territory", Port-Said Engineering Research Journal PSERJ, Published by Faculty of Engineering, Suez Canal University, Port-Said, Egypt, Vol 7, No. 2, September: 559-571.

Bolbol, S.; Hamed, M. and El Sagheer, A. (1997): "Development of a new sets of transformation parameters in Egypt (ETP97) for transforming GPS data into the Egyptian geodetic datum. International Symposium on GIS/GPS , Istanbul , Turkey.
El-Tokhey, M. (1995): "Comparison of some Geopotential Geoid solutions for Egypt", Ain Shams University Scientific Bulletin, Vol. 30, No. 2: 82-101.

El-Tokhey, M. (2000):"On the Determination of Consistent Transformation Parameters between GPS and the Egyptian Geodetic Reference System", Paper presented at the Gravity, Geoid and Geodynamics (GGG2000), Banff, Alberta, Canada.

Hassouna, R. M. (2003): "Modeling of Outer Gravity Field in Egypt using Recent Available Data", Ph.D. Thesis, Department of Civil Engineering, Faculty of Engineering in Shebin El-Kom, Menoufia University, Egypt.

Heiskanen, W.A. and Moritz, H. (1967): "Physical Geodesy", W.H. Freeman and Company, San Francisco and London.

Lachapelle,G. and Tscherning, C.C (1978): "Use of collocation for predicting geoid undulations and related quantities over large areas "The international symposium on the geoid in Europe and Mediterranean area, Ancon, Italy.

Moritz, H. (1973): "Least-squares collocation", Publ. Deut. Geod. Komm. A.

Moritz, H. (1978): "Least-squares collocation", Reviews of geophysics and space physics

Moritz, H. (1980): "Advanced Physical Geodesy", Herbert Wichmann Verlag, Karlsruhe.

Sevilla, M.J.; Gil, A. J. and Sanso, F.(1990):"The Gravimetric Geoid in Spain; first results" Paper presented at the international symposium on the determination of the geoid, Italy, June 11-13.

Smith, D.A. (1998): "There is no such thing as 'The' EGM96 geoid: Subtle points on the use of a global geopotential model", IGeS Bulletin, No. 8: 17-28.

Tscherning, C.C. (1974): "A FORTRAN IV program for the determination of the anomalous potential using stepwise least squares collocation", Report No. 212, Department of Geodetic Science, the Ohio State University.

Tscherning, C.C. (1982a): "Determination of a (quasi) geoid for the Nordic Countries from heterogeneous data using collocation", Paper presented at the meeting of the Nordic Geodetic Commission, Gävle, Sweden, September.

Tscherning, C.C. (1982b): "Geoid-determination in the Norwegian- Greenlandsea. An assessment of recent results", Earth Evolution Sciences, vol, 1, No.2.

Tscherning, C.C. (1993): "An experiment to determine gravity from geoid heights in Turkey", GEOMED Report No. 3.

Tscherning, C.C.; Radwan, A.; Tealeb, A.A.; Mahmoud, S.M.;. Abd El-Monum, M; Hassan, R.; El-Sayed, I. and Saker, K. (2001): "Local geoid determination combining gravity disturbances and GPS/leveling: a case study in the Lake Nasser area, Aswan, Egypt", Journal of Geodesy, 75: 343 - 348.

Wenzel, G. (1998): "Ultra High Degree Geopotential Model GPM3E97A to Degree and Order 1800 tailored to Europe", Paper presented in Proceedings of 2nd Continental Workshop on the Geoid in Europe, March 10-14, Budapest.