THE SIGNIFICANCE OF 3D NETWORK ADJUSTMENT BY USING DIFFERENT LEAST SQUARES METHODS FOR THE CONSTRUCTIONS' MONITORING APPLICATION ON THE MONITORING NETWORK OF THE HOLY AEDICULE IN JERUSALEM

Dimitrios Zachos[1], George Pantazis[2], Evangelia Lambrou[2]

[1]School of Rural and Surveying Engineering, Laboratory of General Geodesy, National Technical University of Athens, Greece (jimizachos@gmail.com)
[2] School of Rural and Surveying Engineering, Laboratory of General Geodesy, National Technical University of Athens, Greece (gpanta@central.ntua.gr, litsal@central.ntua.gr)

Keywords: *Jerusalem; Holy Aedicule; monitoring; minimum constraints; adjustment; ellipsoid; sensitivity; accuracy*

ABSTRACT

This paper deals with the comparison of the minimum constraints of the least squares methods that are used for the adjustment of a 3D monitoring network. The comparison is applied for the 3D geodetic network that was established in the site of the Holy Aedicule of the Holy Sepulchre in Jerusalem during the rehabilitation works (2016 - 2017). The permanent 3D monitoring geodetic network was implemented with special benchmarks. This network was measured at 8 different times from July 2016 to January 2017, in order to allow the displacements' control of the network.

Two methods of minimum constraints adjustment are studied, minimum external and inner constraints. The main difference between these methods is the way each of them overcome the datum deficiency. External constraints require a minimum of known point coordinates and line direction, while inner constraints overcome control problem by using a set of constraints equations.

The network of the Aedicule is being adjusted for every phase, using both methods of minimum constraints. Adjusting the Holy Aedicule network with the external constraints, the accuracy of the determination is better than ±1mm for 95% confidence level, while the absolute and relative displacements are calculated using triaxial ellipsoids. Absolute displacements are equal to 4.1mm. Using inner constraints, the accuracy of determination is better than ±0.5mm for the same confidence level and the absolute displacements equals to 3.8mm.

Through error ellipsoids is proven that inner constraints lead to absolute error ellipsoids 60% smaller than using external constraints, while the displacement vectors do not differ notably, concluding that the sensitivity (i.e. possibility to detect displacements) of the network increases. The results are visually presented with diagrams.

I. INTRODUCTION

The existence and survival of humans in the natural environment is closely related to the constant desire for evolution, which is the timeless driving force for the development of technical works and technology. When it comes for technical works that refer to monuments of international cultural heritage, the need for timely protection is imperative. The Surveyor engineer, who is an integral member of every construction research team, should not forget that the center of all activities is the man and the environment in which he developed as human race. The primary aims of every engineering project should be the safety of human life and the protection of the environment, through sustainable techniques and solutions.

One of the sciences that studies the construction, operation and behavior of technical constructions is Geodesy. The field of geodesy that deals with the monitoring and study of the changes in position, shape and form of technical constructions over time is internationally known as "monitoring" or "displacements/deformations monitoring". Possible changes of a construction are identified with special geodetic and statistical methods, for specific confidence level, in one, two (2D) or three (3D) dimensions. It is more than obvious that deformation analysis is closely connected with the human safety, and therefore the analysis of deformation data has to be carefully interpreted with accuracy and reliability.

Concerning the least squares adjustment of geodetic control networks, it is internationally proposed by the specifications of most countries to use a *minimum constraints adjustment* (*fixed-parameters*). According to ICSM guideline (2014), in Australia and New Zealand, minimally constrained adjustment is the standard, while a fully constrained adjustment is proposed for datum and uncertainty propagation. In US, a minimally constrained adjustment is used for surveys connected to the National Network, as also for GPS networks (Gillins, 2017), while in European countries, such as Croatia (Paar, 2014) fixed-parameters method is also chosen for geodetic control. When it comes to deformation analysis, it is mostly proposed to use a free-net adjustment. For example, Canada (Szostak – Chrzanowski, 2007), Switzerland and Serbia (Sušić, 2017) use inner constraints in a free-net adjustment, while in Malaysia (Setan, 2001) different time epochs are minimally constrained adjusted, and then inner constraints are used in deformation analysis. There are various deformation analysis methods followed, such as robust methods (Danish, M-estimation, LAS, IWST), Karlsruhe method based on independent adjustments, congruency testing methods such as Pelzer's, strain analysis, etc. (Setan, 2001), (Sušić, 2017).

However, these methods are not analyzed here, as this paper studies specifically the different least squares adjustment methods and verifies through extensive comparisons the significance of using inner constraints to adjust monitoring networks, especially when high precision and accurate deformation results are required.

II. THE 3D NETWORK AND THE MINIMUM CONSTRAINTS METHODS

A. *Rank deficiency in 3D geodetic networks*

In general, when a survey network's reference coordinates system cannot be defined due to origin, orientation and/or scale deficiency, the network is called *non-defined*. This *rank deficiency* is the reason of the inability for inverse of the normal coefficient matrix N in the adjustment (i.e., |N|=0), which means that survey observations alone are not sufficient to calculate the coordinates of network's points (Dermanis, 1999).

The maximum rank deficiency of a 3D geodetic network is **7**: unknown coordinates' origin (no fixed points, i.e., the network can be shifted anywhere in 3D space – **3 shifts**), unknown network's orientation (the network can be freely rotated around xy, xz, yz planes – **3 rotations**), unknown scale (the size is not defined as there are no distance measurements – **1 scale** parameter).

However, depending on the available observations, characteristics of the network may be defined. Horizontal angles and slant distances give information for the shape and the scale of the network. If there are azimuths observed, then the direction of y axis is defined, while if zenith angles are measured the direction of z axis is respectively defined. The direction of the local vertical in every point of the network is the same, with no deflection or refraction (for networks of limited size). Thus, zenith angles give information about the orientation of the vertical axis. This means for example, that if there are only observations of distances and zenith angles, then, besides the origin deficiency, there is definition problem in the horizontal geometry of the network (i.e., there is orientation deficiency only in the xy plane) (Kotsakis, 2013).

B. *Minimum Constraints Adjustment Methods*

In order to overcome the rank deficiency problem in geodetic networks, a set of constraints is needed to be imposed. The minimum number of constraints that leads to a solution is equal to the rank deficiency, and such a set of constraints is known as *minimum constraints (MC)* (Dermanis, Rossikopoulos, 1995).

If control stations that contain errors are overweighted, the adjustment will associate the control errors with the observations improperly. This is the reason why most of the times minimum amount of control is chosen, so that observations will need to satisfy the internal geometric constraints of the network only (Ghilani, 2010).

B1. *Fixed - Parameters minimum constraints*

When *fixed – parameters minimum constraints* (also known as *external constraints* by some authors) are imposed in the adjustment, a certain number of parameters of the network (equal in number to the defects) are held fixed.

The origin deficiency is eliminated by fixing one point of the network (e.g., fixed coordinates in arbitrary local coordinate system). For the orientation defect (to the xy plane), it is advisable to hold fixed the direction angle a_{ij} of only one line, which is usually the line between point j and the fixed point i, while additionally when there are neither distance measurements (scale defect), it is necessary to hold two points of the network fixed.

The number **m** of the unknowns when adjusting a geodetic network using *observation equation adjustment*, is equal to the number of point coordinates. However, having fixed parameters, the number m decreases by the number **d** of defects.

B2. *Free-net Adjustment – Minimum inner Constraints*

There is a single MC subset called inner constraints, with the implementation of which there are no fixed parameters in the adjustment. Only when a non-defined network is adjusted using inner constraints, then it is called *free net*, as every point is free to move in space.

Such a set of MC *minimize the trace* of the covariance matrix $\hat{V}_{\hat{x}}$, meaning that it yields minimum variances of the adjusted quantities (diagonal elements of $\hat{V}_{\hat{x}}$), while the adjustment also yields to coordinates (*a.k.a. free-net coordinates*) such that the corrections dx, comparing with the initial approximations, are minimum (Tan, 2005).

The rank deficiency problem is now encountered with inner constraints equation, without fixed parameters, depending on the defects. For the general case of a 3D geodetic network with 7 defects, the inner constraints equations are expressed as:

- For the definition of the reference system origin:

$$\sum_{i=1}^{v} dx_i = \sum_{i=1}^{v} (\hat{x}_i - x_i^o) = 0 \qquad (1a)$$

$$\sum_{i=1}^{v} dy_i = \sum_{i=1}^{v} (\hat{y}_i - y_i^o) = 0 \qquad (1b)$$

$$\sum_{i=1}^{v} dz_i = \sum_{i=1}^{v} (\hat{z}_i - z_i^o) = 0 \qquad (1c)$$

- For the orientation of the reference system:

$$\sum_{i=1}^{v} (z_i^o \cdot dy_i - y_i^o \cdot dz_i) = 0 \qquad (2a)$$

$$\sum_{i=1}^{v} (x_i^o \cdot dz_i - z_i^o \cdot dx_i) = 0 \qquad (2b)$$

$$\sum_{i=1}^{v} (y_i^o \cdot dx_i - x_i^o \cdot dy_i) = 0 \qquad (2c)$$

where equations 2a, 2b, 2c refer to rotations about yz, xz, xy plane respectively.

- For the definition of the network's size (unknown scale):

$$\sum_{i=1}^{v} (x_i^o \cdot dx_i + y_i^o \cdot dy_i + z_i^o \cdot dz_i) = 0 \qquad (3)$$

where v = the number of network's points
$\hat{x}_i, \hat{y}_i, \hat{z}_i$ = the adjusted coordinates
x_i^o, y_i^o, z_i^o = their initial approximated values
dx_i, dy_i, dz_i = the corrections of the unknowns, which are the elements of vector dx

It is proven that inner constraints are such that the ***centroid** of the network does not change*, concerning to its position, orientation and average distance from all points (Agatza-Balodimou, 2018). This means that the center of the approximated coordinates is the same as the center of the adjusted free-net coordinates.

There are two ways to adjust the network with inner constraints. a) Considering initial approximated coordinates as they are given, but also b) importing initial approximated coordinates *referring to the centroid* of the network. The centroid (K) is not necessarily a surveyed point, but it is defined as the average of the coordinates. In this way, the coordinate reference system of the approximated coordinates is shifted so that the origin is the same as the centroid.

Furthermore, the *residuals u* and the *a posteriori reference standard deviation* $\hat{\sigma}_o$ are independent of the adjustment method choice, as soon as the constraints are minimum.

C. *Displacements control in 3D space*

In a deformation survey, after adjusting the network, *triaxial error ellipsoids* are used for absolute and relative displacements detection, since displacements control takes place in 3D. The elements of error ellipsoids are easily obtained by calculating the characteristic elements of every $\hat{V}_{dx}$ and $\hat{V}_{d\Delta x}$ sub-matrix. The *length* of every semi-axis of ellipsoids is equal to the square root of *eigenvalues*, while *eigenvectors* give information about the *orientation* of the ellipsoids (i.e., components of the unit vectors $\vec{u}, \vec{v}, \vec{w}$ to the x,y,z axis, where u,v,w are the semiaxis of the ellipsoid) (Agatza-Balodimou, 2018), (Bektaş, 2015).

Error ellipsoids are also modified in this paper for 95% confidence level by using the multiplier parameter c, which is computed using *F-statistic*, since ellipsoids' elements are calculated using the a posteriori covariance matrix:

$$c = \sqrt{3 \cdot F_{p,3,r}} \qquad (4)$$

where p = confidence level
r = degrees of freedom

The displacements' control follows three steps. At the beginning, for every ellipsoid it is checked if the displacement vector (dr) is bigger than the maximum semiaxis of the ellipsoid (dr > $\sigma_{max} \cdot$c). After that, the graphic check follows, by drawing the error ellipsoids and the displacement vectors, while the 3rd step, which is mathematically the most reliable, uses the ellipsoid equation to control the displacements. Specifically, it is considered that there is absolute or relative displacement, if the following equation is true:

$$\frac{u^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2} + \frac{w^2}{\sigma_w^2} > 1 \qquad (5)$$

where u, v, w are replaced with the components of displacement vector and $\sigma_u, \sigma_v, \sigma_w$ are the lengths of ellipsoid semiaxis (Gendzwill, 1981).

III. APPLICATION ON THE 3D MONITORING NETWORK OF THE HOLY AEDICULE

A. *Description of the network*

For the extensive monitoring of the Holy Aedicule, in order possible displacements to be detected, but also for all the high precision geodetic measurements and the documentation of the monument, a three -

dimensional geodetic control network of **v=13 points** was established, in arbitrary local coordinates system. The permanent points of the network were implemented with special benchmarks.

The network was measured in 8 different time intervals, in order to allow the displacements' control of the network. In every time phase distances, horizontal and zenith angles were observed, using a suitable integrated geodetic total station of ±1'' angular accuracy (based on DIN 18723) and ±1mm±2ppm distance accuracy (according to ISO17123-4) (Pantazis, Lambrou, 2017), (Moropoulou et al, 2018). The geometry of the 3D geodetic network is shown in Figure 1.

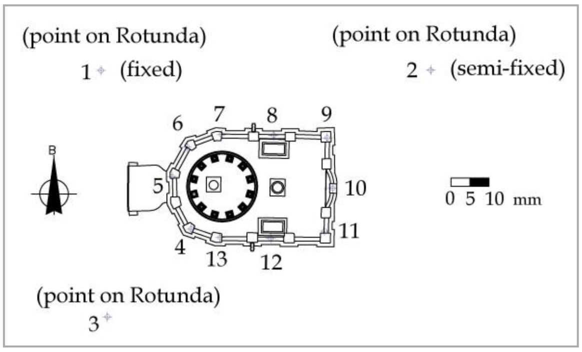

Figure 1. The geometry of the 3D control network.

Since distances, horizontal and zenith angles were observed, the rank deficiency of the network is equal to **4**. Specifically, the coordinates' origin (3 shifts) and the orientation in the xy plane (1 rotation) are not defined.

B. *Comparison of minimum constraints methods*.

B1. *With regard to the* ***adjustment process***

The main difference between the two MC methods is the way each of them deals with the network's rank deficiency. Depending on the number and type of the defects, their elimination is achieved by *holding certain parameters of the network fixed*, using *fixed-parameters* method, or by introducing *appropriate constraint equations* in the adjustment, using *inner constraints* method.

Hence, when using inner constraints more computations are required. This is because all network's points are considered unknown, and so there are no coefficients to be deleted in the observation equations, to form the design matrix A (in contrast with the coefficients of the fixed parameters in the fixed-parameters method), but also because of the input of a constraint matrix *C* and a pseudoinverse N^+ in the computations.

Moreover, the problem of initial approximated coordinates is raised. When parameters of the network are held fixed, approximated coordinates can easily be calculated using basic geodetic computations. However, this is not possible in a free-net adjustment, as there are only unknown points. This is why using inner constraints requires a *set of known coordinates from prior adjustment* of the network, in order to be imported as the initial approximated ones for the free-net adjustment. As explained, these initial coordinates can be either transformed to refer to the centroid of the network or not. The Aedicule's network was adjusted using both ways (same results), although it is preferable to choose *centroid referred coordinates* in order to avoid numerical problems and simplify computations (especially for N^+ pseudoinverse matrix).

By theory, inner constraints implement the reference system which minimizes the trace of the final covariance matrix $\hat{V}_{\hat{x}}$, leading to minimum variances for the point coordinates, and so to smaller error ellipsoids, in contrast to every other subset of MC, which is the great advantage of the method.

B2. *With regard to the* ***results of the adjustments***

The 3D geodetic network of the Holy Aedicule was adjusted for every time phase using both methods of MC. In addition, global test (Chi-square) and local test (*data snooping* of Baarda method) were used for error detection and check of the reliability of the network. However, the aim of the paper is not to analyze these tests.

For a set of observations that successfully passes the Baarda test for a percent of 95.8%, points' coordinates are computed using *fixed-parameters* adjustment with an accuracy better than **±1mm** for 95% confidence level, while using *inner constraints* this accuracy is better than **±0.5mm**.

In order to compare the results of two adjustment methods, the difference of the coordinates $(x,y,z)_{free\text{-}net} - (x,y,z)_{fixed\text{-}param.}$ was calculated for every phase, as for the accuracies respectively, as shown in Figures 2 and 3.

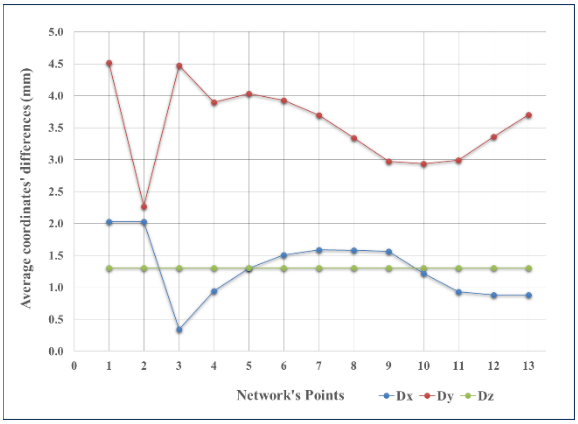

Figure 2. Average coordinates' differences (mm)

The y coordinates are the ones with the maximum deviations, which fluctuates from 3.5mm to 4.0mm, with an average value of 3.6mm, while an important percent of 23% amounts to 4.0 - 4.5mm. The x and z coordinates differ for 1.3mm between the two methods, with no notable differences for the x,y,z accuracies (±0.2mm).

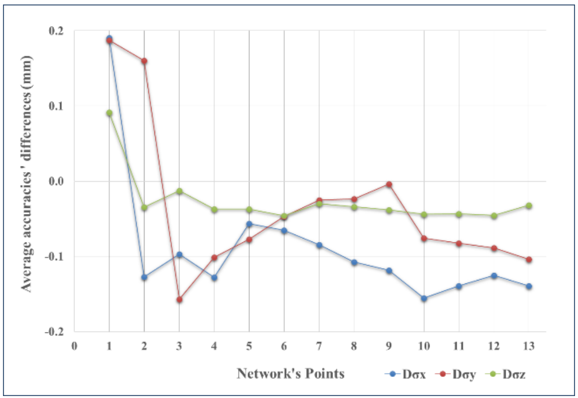

Figure 3. Average accuracies' differences (mm)

B3. *With regard to the* ***displacements control***

One of the main purposes of the network's establishment was the displacements control of the monument. Through triaxial error ellipsoids, possible displacements are studied in 3D space, giving in this way different results than studying these displacements in horizontal and vertical direction separately. The check in this way becomes stronger and more sensitive.

For the comparison of the two MC methods, regarding to the results of absolute and relative position change of the benchmarks for displacements control, the *size of the displacements vectors* and the *size of absolute and relative error ellipsoids* are compared. As calculated, there is no considerable difference between the vectors of **absolute** position change, which amounts to **4.1mm** for *fixed-parameters* and **3.8mm** for *free-net* adjustment on average, although the first ones are smaller from free-net vectors in a percent of 72%. The vectors of **relative** position change are equal for the two methods, with a mean value of **5.5mm**.

More interest appears in the comparison of the size of absolute and relative error ellipsoids, formed for the displacements control of the network, through which the theory of inner constraints for minimum point variances is verified. For this comparison, the volumes of absolute and relative error ellipsoids are computed for the benchmarks of the network, using the geometric equation for ellipsoid's volume:

$$V_{ellipsoid} = \frac{4}{3} \cdot \pi \cdot r_1 \cdot r_2 \cdot r_3 \qquad (20)$$

where $r_1 = \sigma_u$, $r_2 = \sigma_v$ and $r_3 = \sigma_w$.

As shown in Figures 4 and 5, the displacement checks of free-net adjustments lead in total to sizably smaller **absolute** error ellipsoids than the ones of fixed-parameters, with inner constraints' ellipsoids ***60% smaller in volume*** than fixed-parameters' ellipsoids. Furthermore, these absolute ellipsoids are more homogenous, with the same geometrical characteristics.

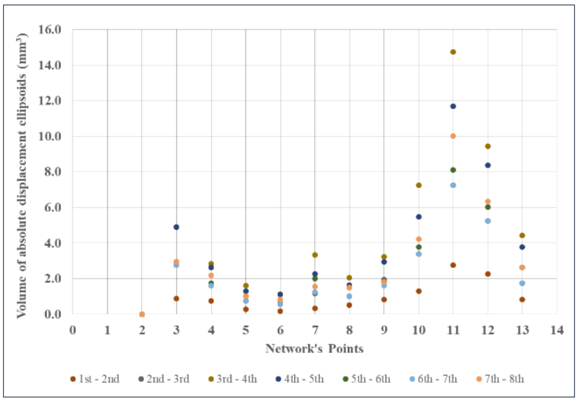

Figure 4. Volume of absolute displacement ellipsoids (mm³) for fixed-parameters adjustments.

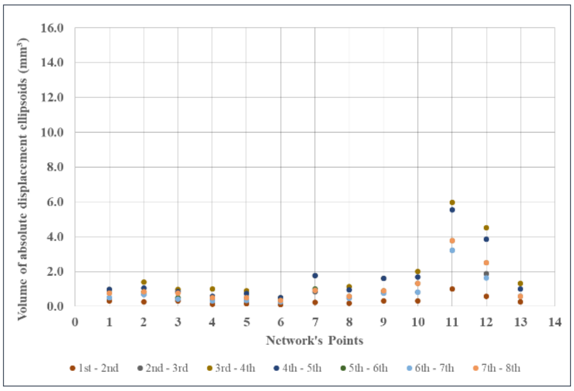

Figure 5. Volume of absolute displacement ellipsoids (mm³) for free-net adjustments.

With the same process, the volume of error ellipsoids of **relative** displacements between the benchmarks of the network are compared. After extensive data analysis, it is computed that **68%** of relative ellipsoids, considering a *free-net*, are smaller than the ellipsoids of *fixed-parameters*. Specifically, these relative ellipsoids, using inner constraints, are *smaller in volume in a percent of **13.5%***. For the greatest understanding and evaluation, the results are shown in the following Figures.

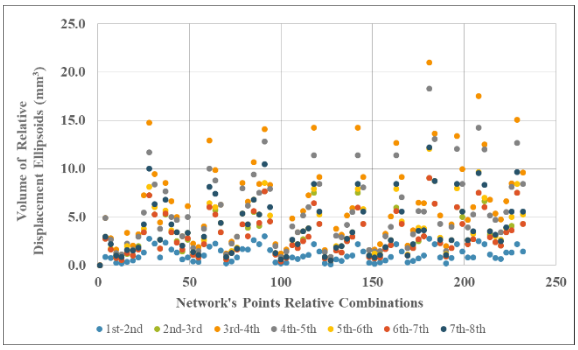

Figure 6. Volume of relative displacement ellipsoids (mm³) for fixed-parameters adjustments.

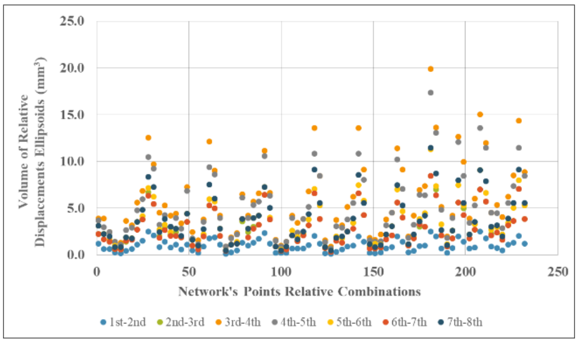

Figure 7. Volume of relative displacement ellipsoids (mm³) for free-net adjustments.

Although the difference may not be significant, it does not cease to improve the sensitivity of the network.

III. CONCLUSIONS

An extensive comparison of the two possible minimum constraints methods of adjustment in geodetic networks has been carried out in this paper, while also free-net adjustment is fully analyzed in order to shed light on some dark points on the topic of inner constraints. Although the theory is known, it is very interesting and challenging to verify it by observing how a 3D geodetic monitoring network behaves when inner constraints are used, as most of the contemporary adjustment geodetic programs use fixed-parameters solution to adjust survey networks.

In first stage, the *success of such a 3D geodetic network to detect crucial displacements, with an accuracy better than ±1mm for 95% confidence level*, is noted. The special implementation method of the benchmarks and the permanent enforced installation of the geodetic instrument, which eliminates the centering and levelling errors, in combination with a special methodology for the height measurement of the total station, with an accuracy of ±0.3mm, were of catalytic importance for the ability of the network to detect displacements of such size.

After application of MC adjustment methods in the Holy Aedicule's 3D network, it is shown that first of all *inner constraints give coordinates of better accuracy (±0.5mm)*. As mentioned above, when using inner constraints, known coordinates are required, in order to be imported as approximated for the adjustment. These initial coordinates affect the adjusted *inner coordinates*, because they define the location and orientation of the adjusted free net. Different approximated coordinates lead to different solution and obviously when it comes for a deformation survey in different time intervals, the input approximated coordinates should be the same for every adjustment. In addition, a free-net adjustment gives the *minimum overall movement for the points of the network*, and that is because it gives the solution with which the points best fit with the observations and the initial approximated coordinates (Harvey 1998, Kotsakis 2013).

From the comparison of the adjusted coordinates of each method, there are *considerable differences of 3.6mm for y coordinates and 1.3mm for x and z coordinates*. This is understandable if the accuracy of the adjustment is taken in mind.

As proven from the results for the Holy Aedicule's network, for absolute displacement vectors with no significant difference, *free-net adjustment produces absolute displacement ellipsoids of volume* ***60%*** *smaller than the one of fixed-parameters*. Relative displacement ellipsoids are also affected, but to a lesser extent, as they are *smaller in volume for a percent of* ***13.5%*** *using inner constraints.* In this way, there is a *remarkable improvement of the* ***sensitivity*** *of the network*, i.e., its ability to detect displacements, especially absolute displacements.

It is also noted that, although fixed-parameters adjustment can be implemented in all types of networks (irrespective of size), a free net choice is better for high precision and/or deformation monitoring surveys in local datums, for networks of limited size.

There are two main conclusions exported from this paper. When it comes for 3D geodetic networks, *is better for the displacements control to be studied in 3D* using triaxial ellipsoids, in order to have a more complete and reliable view of the kinematic behavior of the points, as the control becomes in this way stronger (more sensitive).

Depending also on the type of the network and the purpose of each geodetic survey, the significance of using *inner constraints in a free-net adjustment* is highlighted, because with this method an *approach closer to the reality* is achieved. Especially for high precision and deformation surveys, where a few (mm) can be critical, it is rarely guaranteed that there are stable points of the network, so that can be held fixed its control. By this mean, every point should be considered to move in space, unreliable to be the fixed one for the adjustment. Over and above this fact, the sensitivity of the network increases sizably, detecting displacements with reliability and accuracy.

IV. ACKNOWLEDGEMENTS

The study and rehabilitation project of the Holy Aedicule were carried out under the governance of His Beatitude Patriarch of Jerusalem, Theophilos III.

The Common Agreement of the Status Quo Christian Communities provided the statutory framework for the execution of the project; His Paternity the Custos of the Holy Land, Archbishop Pierbattista Pizzaballa (until May 2016 – now the Apostolic Administrator of the Latin Patriarchate of Jerusalem), Fr. Francesco Patton (from June 2016), and His Beatitude the Armenian Patriarch of

Jerusalem, Nourhan Manougian, authorized His Beatitude the Patriarch of Jerusalem, Theophilos III, and NTUA to perform this investigation and project.

The interdisciplinary NTUA team for the Protection of Monuments, Em. Korres, A. Georgopoulos, A. Moropoulou, C. Spyrakos, Ch. Mouzakis, were responsible for the rehabilitation project and A. Moropoulou, as Chief Scientific Supervisor, was responsible for its scientific supervision.

References

Agatza – Balodimou, A.M., Panou, G. (2018). Least Squares Method and Applications, *Tziola puplications*, Thessaloniki

Bektaş, S. (2015). Geodetic Computations on Triaxial Ellipsoid, In: *International Journal of Mining Science*, Vol. 1, issue 1, pp. 25-34

Chrzanowski, A., Chen, Y.Q., Secord, J.M., (1986). Geometrical analysis of Deformation Surveys, *Deformation Measurements Workshop*, University of New Brunswick, Canada

Dermanis, A., Rossikopoulos, D., Fotiou, A. (1995). Surveying Networks and Computations: computer programs and examples, *Ziti publications,* Thessaloniki

Dermanis, A. (1999). Observation Adjustments and Theory of Estimation, *Ziti publications*, Vol. 2, Thessaloniki

Gendzwill, D. J., Stauffer, M. R. (1981). Analysis of Triaxial Ellipsoids: Their Shapes, Plane Sections, and Plane Projections, In: *Journal of the International Association for Mathematical Geology*, Vol. 13, No. 2, pp. 135-152

Ghilani, C. (2010). Adjustment Computations: spatial data analysis, *John Wiley & Sons, Inc.*, New Jersey

Gillins, D. T., Dennis, M. (2017). Inclusion of Leveling with GNSS Observations in a Single, 3-D Geodetic Survey Network Adjustment, In: *conference paper for FIG working week*, Helsinki, Finland

Harvey, B.R., Elford, D., Turner, C. (1998). Calculation of 3D Control Surveys, In: *The Australian Surveyor*, Vol. 43, issue 2, pp. 109-118

ICSM Permanent Committee on Geodesy, (2014). Guideline for the Adjustment and Evaluation of Survey Control, special publication 1, Version 2.1, Commonwealth of Australia

Kotsakis, C. (2013). Generalized inner constraints for geodetic network densification problems, In: *Journal of Geodesy*, Vol. 87, issue 7, pp. 661-673

Moropoulou Antonia, Karoglou Maria, Agapakis Ioannis, Mouzakis Charalambos, Asimakopoulos Simos, Pantazis George, Lambrou Evangelia (2018). Holy Selphuchre Monitoring System. 1st International Conference on "Transdisciplinary Multispectral Modelling and Cooperation for the Preservation of Cultural Heritage (TMM_CH)", Athens**.**

Paar, R., Novaković, G., Zulijani, E., (2009). Positioning Accuracy Standards for Geodetic Control, University of Zagreb, Faculty of Geodesy

Pantazis, G., Lambrou, E. (2017). Monitoring of the kinematic behavior of the Holy Aedicule of the Holy Sepulchre in Jerusalem during the rehabilitation works, *presentation in 5th national conference of Rural and Surveying Engineers*, Athens

Setan, H., Singh, R., (2001). Deformation analysis of a geodetic monitoring network, *Geomatica*, Vol. 55, No. 3, Malaysia

Sušić, Z., Batilović, M., Ninkov, T., Bulatopić, V., Aleksić, I., Nikolić, G., (2017). Geometric Deformation analysis in Free Geodetic Networks: Case Study for Fruška Gora in Serbia, *Acta Geodyn. Geomater.,* Vol. 14, No. 3 (187), pp. 341-355

Szostak – Chrzanowski, A., Massiéra, M., Chrzanowski, A., (2007). Analysis of deformations of large earth dams, In: *Journal of Applied Geodesy jag*, Vol. 1, issue 2, pp. 81-89

Tan, W. (2005). Inner Constraints for 3-D Survey Network, In: *Journal of Spatial Science*, Taylor and Francis Ltd, Vol. 50, No. 1, pp. 91-94

Zachos, D. (2018). Comparison of the minimum constraints least squares adjustment methods in a 3D monitoring network. Analysis of successive monitoring data. Application on the network of the Holy Sepulchre of the Holy Tomb, *Diploma Thesis*, NTUA, Athens