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MONITORING AND ANALYSIS OF ROCK BLOCKS DEFORMATIONS

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Abstract: Mass movements of sandstone rock blocks in the Table Mountains National Park (SW Poland) generate threat for intensive tourist activity there. In the 70'ties of the 20th Century a monitoring and measurement system has been set up for monitoring and quantitative assessment of this phenomenon. In the result of long-term satellite GPS and geodetic (Total Station, precise leveling) surveys and relative measurements with crack gauges spatial displacement vectors have been determined for points located on these rock blocks as well as their relative movements calculated. These results have been used to calculate deformation parameters (translation, rotation, shear strain) of the analyzed rock blocks. This research has been described on the example of the most mobile part of the Szczeliniec Massif in the largest rock crack "Piekielko"(Little Hell).

1. INTRODUCTION

Natural and man-made changes occurring in the earth's crust have different nature and manifest through violent reactions as earthquakes and slow reactions such as mass movement. Deformations of the edge zone of cretaceous sandstones in the Szczeliniec Massif in Table Mountains National Park (SW Poland) can be counted among these phenomena. The Massif reaching 919 m above sea level is the highest part of the Table Mountains. Rock formations shaped in geological times, as a result of water and wind erosion, as well as, other factors are a tourist attraction. Deformations of rock blocks may initiate potential danger for tourist activity.

Monitoring using special control-measurement system (Cacon, Kontny, 1993) started in the 70-ties of the 20th Century to quantitatively assess deformation of rock blocks. The results of cyclic satellite GPS, geodetic (precise levelling and Total Station), as well as, relative (crack-gauge) measurements produced vectors of movements for points located on blocks of rock and relative movements of these blocks. The results of measurements covering the whole Szczeliniec Massif have been presented in paper by Cacon and others (2008).

In this work the results of geodetic and relative measurements of rock block movement in the most mobile part of the Szczeliniec and the largest cleft "Piekielko" are characterised. These results have been used to calculate deformation, translation, rotation and linear strain parameters of the analysed rock blocks.

2. CHARACTERISTICS OF MEASUREMENT RESULTS OF ROCK BLOCKS MOVEMENT IN THE “PIEKIELKO”

The measurements of rock blocks movements in the “Piekielko” are carried out in the II and III control-measurement segments. Geodetic observations (segment II) are done in a network of points (Fig. 1) fixed with metal pins in places suitable for angular-linear and levelling measurements. The network has been connected to regional satellite-gravimetric network “Table Mountains” (segment I) through point 112 (Cacon et al., 2003). Since the beginning of this research in 1982 to year 2006, 13 measurement cycles have been done. Geodetic measurements (horizontal) had been carried out in the beginning with theodolite and millimetre scale metal tape replaced later with invar tape. Since 2000 observations are carried out using Total Station.

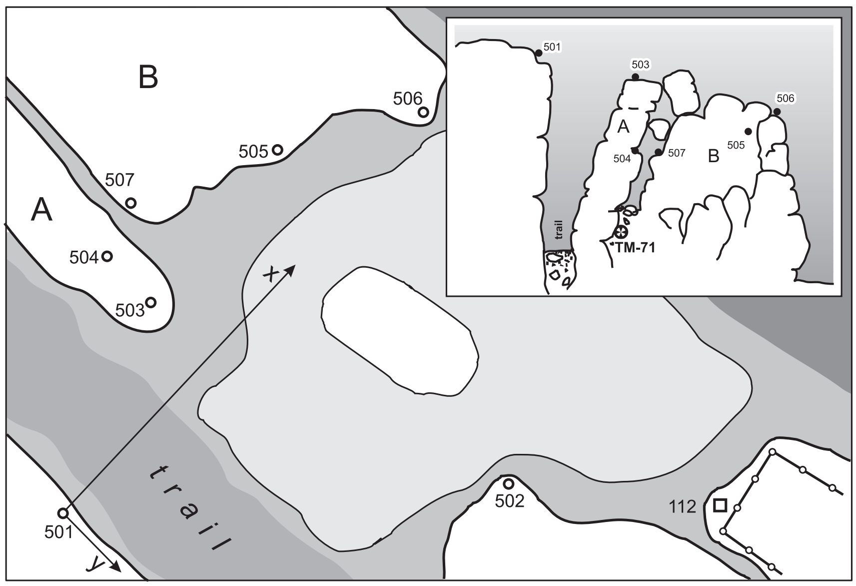

Fig. 1 - Sketch of the “Piekielko” network

Height measurements have been realised with levelling instrument and precise measuring rods. Accuracies of height and horizontal measurements before 2000 felt within the ±(0.1–5.0) mm range and after 2000 within the ±(0.1–2.0) mm range.

The greatest spatial movements in the 1992–2006 period were registered on points 503 (38.2 mm) and 507 (36.4 mm). Point 507 experienced the greatest subsidence (-25.0 mm), which corresponds to a velocity of 1 mm/year. Vectors of horizontal displacements in a profile perpendicular to a tourist trail in relation to point 501 (inside the Massif) have been shown in Fig. 2 and 3.

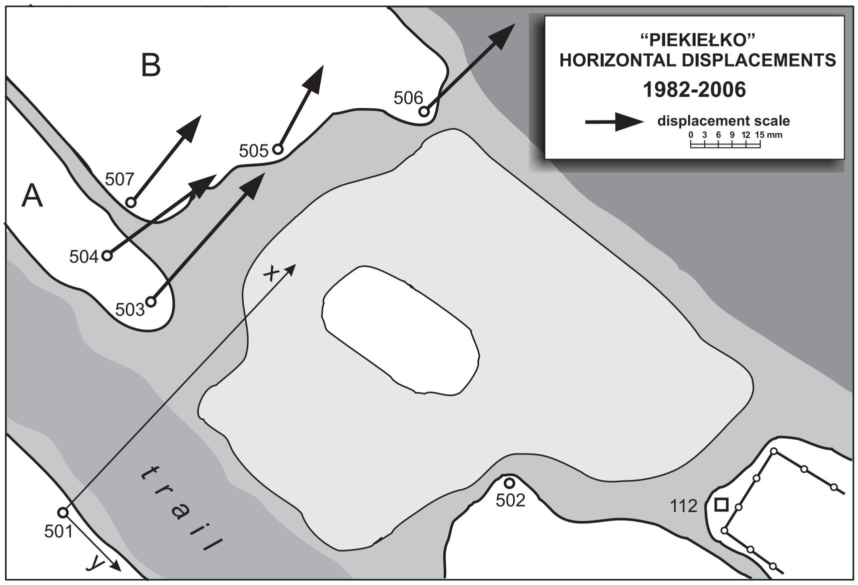

Fig. 2 - Horizontal movements of points in the "Piekielko" network

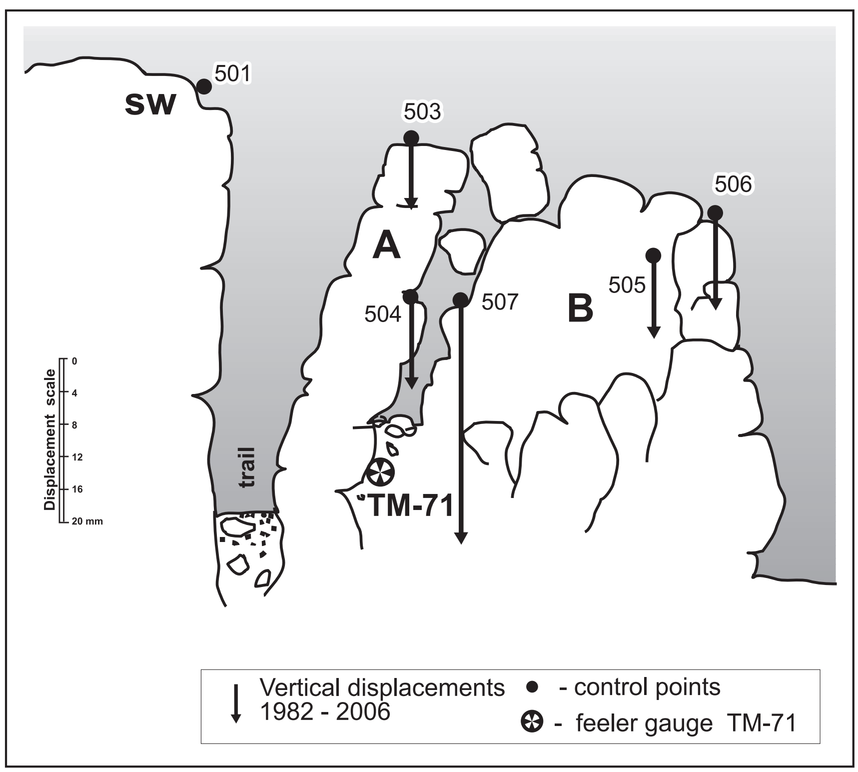

Fig. 3 - Vertical movements of in the "Piekielko" network

Relative spatial movements (segment III) have been registered in a local (x, y, z) system since 1979 with TM-71 crack-gauge (Kostak 1991). The instrument is installed in the lowest place between rock block A and block B. Crack-gauge observations carried out monthly provide accuracy of ±0.05 mm. It must be noted that the results of the first three years of crack-gauge observations (1979–1982) laid foundations for organising geodetic measurements of rock blocks movements (segment II). The results of relative movements of blocks A and B have been shown in Fig. 4.

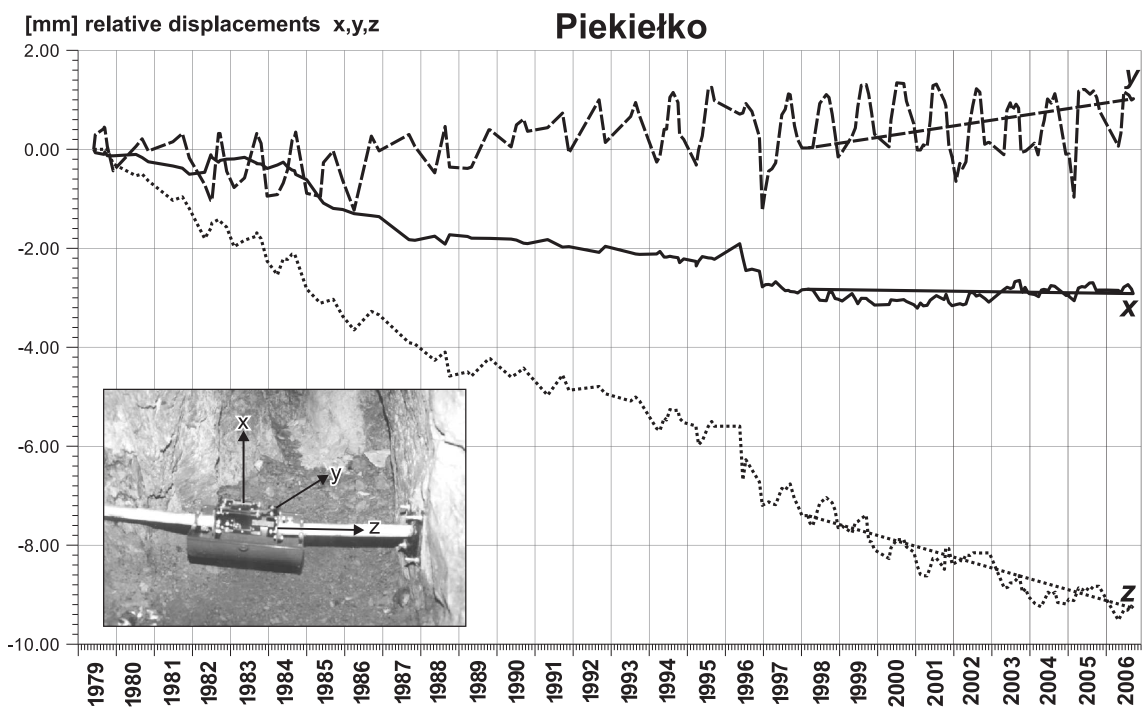

Fig. 4 - Results of relative movements of rock blocks

3. DISPLACEMENT AND SHEAR PARAMETERS OF A SOLID

Geodetic measurements in deformation monitoring of engineering or natural objects provide information on relative motion of discrete measured points. Kinematics of an object (solid) as a continuous medium is described usually by means of parameters of a vector field approximated from discrete points (*inter alios* Vaniček, Krakivsky, 1986). In geometrical interpretations usually strain tensor parameters of the adopted deformation model are used (Schneider, 1982). These parameters can be determined from relationship between displacement vectors and strain tensor parameters of the model adopted. In many local applications the analyses are reduced to a two-dimensional strain problem in a Cartesian coordinate system. Extensive review of such applications is given by Schneider (1982). Analyses related to global or regional scales require application of curvilinear coordinate systems tied to surface of the earth's ellipsoid (Altiner, 1999; Voosoghi, 2000).

As the fundamental relations between displacement vector and strain tensor are universally known from continuous medium mechanics, only the most frequently used, in geodetic literature, deformation measures are reminded here (e.g. Altiner, 1999; Schneider, 1982; Szostak-Chrzanowski et al., 2006).

Deformation tensor parameters are calculated by factoring gradient of the displacement function:

$$u = u(x) = u_0 + \frac{\partial u(x)}{\partial x} dx + ... \cong t + J \cdot x \tag{1}$$

into symmetrical and anti-symmetrical components:

$$J = \frac{1}{2}\left(J + J^T\right) + \frac{1}{2}\left(J - J^T\right) = E + \Omega \tag{2}$$

Therefore displacement vector can be expressed by combination of translation (t), rotation (Ω) and strain (E) of the object

$$u = u(x) = t + \Omega \cdot x + E \cdot x \tag{3}$$

The t vector contains components of the object's translation $t^T = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}$.

In 3D space symmetrical strain tensor E and anti-symmetrical rotation tensor Ω can be expressed in the form of:

$$E_{xyz} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}, \tag{4}$$

$$\Omega_{xyz} = \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} - \frac{\partial u_z}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) & 0 \end{bmatrix}. \tag{5}$$

Elongation components of the strain tensor characterise linear strains in directions of the coordinate system's axis, whereas shear components characterise non-dilatational strain. Strain tensor values make-up principal strains ε in directions indicated by eigenvectors. For a two-dimensional case:

$$\varepsilon = SDS^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \tag{6}$$

From factoring strain tensor E into conformal and anti-conformal components we obtain:

$$E = \begin{bmatrix} \delta+\tau & \nu-\varpi \\ \nu+\varpi & \delta-\tau \end{bmatrix} = \begin{bmatrix} \delta & -\varpi \\ \varpi & \delta \end{bmatrix} + \begin{bmatrix} \tau & \nu \\ \nu & -\tau \end{bmatrix}, \tag{7}$$

where δ denotes linear dilatation (average unit elongation), ϖ is mean angle of rotation, whereas τ and ν are shear tensor coefficients (Schneider, 1982).

Vector of translation and tensor of rigid rotation describe rigid displacement of a solid, while strain tensor describes its change of shape. Some basic parameters of state of strain can be reduced to one-dimensional problem and calculated directly from the following definitions (Burchfiel, 2005; Hjelmstad, 1997):

- elongation (engineering strain)

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}, \tag{8}$$

- stretch

$$\lambda = \frac{l}{l_0} = 1 + e, \tag{9}$$

Lagrange (Green) strain

$$E = \frac{1}{2}\left(\frac{l^2 - l_0^2}{l_0^2}\right) = \frac{1}{2}\left(\lambda^2 - 1\right), \tag{10}$$

- Euler (Almansi) strain

$$e = \frac{1}{2}\left(\frac{l^2 - l_0^2}{l^2}\right) = \frac{1}{2}\left(1 - \frac{1}{\lambda^2}\right), \tag{11}$$

- dilatation (change in volume or area)

$$\Delta = \frac{\Delta V}{V_0} = \frac{V - V_0}{V_0}. \tag{12}$$

4. CALCULATION OF DEFORMATION PARAMETERS FOR SELECTED ROCK BLOCKS IN THE “PIEKIELKO”

Adjusted coordinates of points located on rock blocks A and B (Fig. 1). converted to a local XYZ coordinate system form the basis for calculating deformation parameters of these blocks. Orientation of the local system is adjusted to geometry of main rock structures in edge zone of the massif – X axis is perpendicular to main edge of massif while Y axis is in conformity with course of the largest rock cleft and tourist trail. The deformation parameters of blocks have been calculated for period between the first (1982) and the last (2006) geodetic measurement of the control micro-network.

The reference frame is defined by points: 501, 502 and 112. Fixed positions of these points in relation to each other have been confirmed by analysing changes of geometric components between them (distances, angles and height differences). Displacement values with accuracies for “Piekielko” Network points are given in Tab. 1.

Point	ux (mm)	uy (mm)	uz (mm)	mux (mm)	muy (mm)	muz (mm)
501	0.0	0.0	0.0	0.0	0.0	0.0
112	-1.5	-1.5	-0.1	2.4	2.4	1.4
502	-2.1	-1.1	0.7	2.9	2.9	1.4
503	34.3	-1.5	-7.1	4.3	4.3	1.4
504	24.3	6.9	-7.0	4.6	4.6	1.4
505	14.7	-5.2	-7.4	4.6	4.6	1.4
506	20.5	0.6	-9.3	4.8	4.8	1.4
507	22.8	2.2	-21.3	6.7	6.7	1.4

Tab. 1 - Displacements of the “Piekielko” Network points for period 1982–2006

Relatively large values of elongation are the consequence of small distances between measured points.

Any linear model related to deformation parameters *p* can be written generally in the form:

$$u + v = B \cdot p \,, \tag{13}$$

where *u* denotes displacement vector of points and *v* residua vector of the model. When solved using the least squares method it provides estimator vector of the unknown parameters:

$$p = \left(B^T C_u^{-1} B\right)^{-1} B^T C_u^{-1} u \,, \tag{14}$$

with covariance matrix of the unknown parameters:

$$C_p = \hat{\sigma}_0^2 (B^T C_u^{-1} B)^{-1} \,. \tag{15}$$

The model can be accepted as adequately (in statistical sense) describing deformations of an object if the global test is passed:

$$M = v^T C_u^{-1} v = \frac{\hat{\sigma}_0^2}{\sigma_0^2}(n-k) > \chi^2_{1-\alpha,f} \,. \tag{16}$$

If the test fails one can eliminate points of the model that fail the local test:

$$m_i = \frac{\left|\hat{v}_i\right|}{\sigma_{\hat{v}_i}} \le N_{\alpha/2}\left(0,1\right). \tag{17}$$

The accepted model of deformation will describe reality the better the smaller are the residua and the M test accepted with higher level of confidence.

In accordance with (3)–(5) the model (13) of rigid body deformation of blocks A and B can be generally written as:

$$\begin{bmatrix} v_A \\ v_B \\ v_{AB} \end{bmatrix} = \begin{bmatrix} 1 & 0 & S_A & 0 \\ 0 & 1 & 0 & S_B \\ -1 & 1 & -S_A & S_B \end{bmatrix} \begin{bmatrix} t_A \\ t_B \\ \varpi_A \\ \varpi_B \end{bmatrix} - \begin{bmatrix} u_A \\ u_B \\ u_{AB} \end{bmatrix}, \qquad (18)$$

where sub-matrix S for each point comes to:

$$S_i = \begin{bmatrix} -(y - y_{0i}) & 0 & (z - z_{0i}) \\ (x - x_{0i}) & -(z - z_{0i}) & 0 \\ 0 & (y - y_{0i}) & -(x - x_{0i}) \end{bmatrix}, \qquad (19)$$

while sub-vectors of translation and rotation parameters of blocks are $t_i^T = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}$, $\varpi_i^T = \begin{bmatrix} \varpi_{xy} & \varpi_{yz} & \varpi_{xz} \end{bmatrix}$, and components of measured displacements of points $u_i^T = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}$. Rotation point of each block (centre of rotation) has coordinates $X_{0i}^{\ T} = \begin{bmatrix} x_{0i} & y_{0i} & z_{0i} \end{bmatrix}$.

The third line in model (18) concerns relative observations (TM-71 crack-gauge) between points located on both rock blocks A and B (Fig. 3). Parameters of translation and rotation of both blocks calculated for the 1982–2006 period are given in Tab. 3.

BLOCK	PARAMETR	VALUE	RMS	RATIO
A	t_x (mm)	0.1	5.7	0.0
	t_y (mm)	-0.4	4.9	-0.1
	t_z (mm)	-6.7	0.8	-
	ϖ_{xy} (rad)	-0.000191	0.001594	-0.1
	ϖ_{xz} (rad)	0.001364	0.000331	4.1
	ϖ_{yz} (rad)	-0.000114	0.000270	-0.4
x.B	t_x (mm)	25.1	6.2	4.1
	t_y (mm)	9.0	9.7	0.9
	t_z (mm)	-12.1	0.8	-15.7
	ϖ_{xy} (rad)	0.000001	0.000577	0.0
	ϖ_{xz} (rad)	-0.000285	0.000291	-1.0
	ϖ_{yz} (rad)	0.000440	0.000513	0.9

Tab. 3 - Translation and rotation parameters of blocks A and B

The value of global congruency test of the model (16) is:

$$M = 6.79 < \chi^2_{0.95,6} = 12.59,$$

Therefore there are no reasons to reject this model. Local test (17) at α=0.05 confidence level fails for two values of residua only, and at α=0.01 confidence level only one value – u_z component of point 505.

$$m_i = 2.69 > N_{\alpha/2}(0,1) = 2.57\ .$$

Distribution of residua has been shown in Fig. 5.

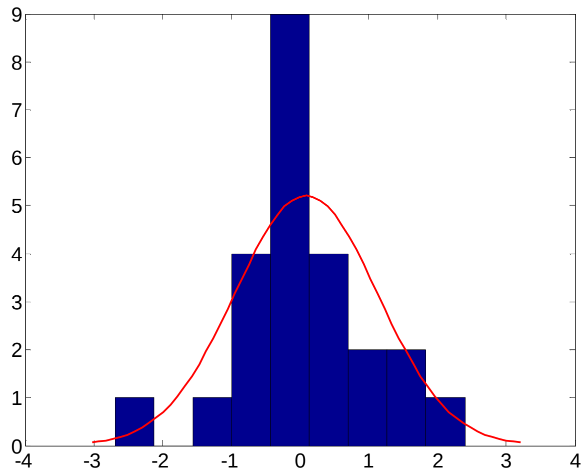

Fig. 5 - Standardized residua’s distribution of the rigid body deformation model

The character of deformations of the rigid blocks A and B has been pictured schematically in Fig. 6. Block A subsides at a rate of approx. 0.3 mm/year and tilts towards slope of the massif at a rate of 0.000057 rad/year (about 11.6’’/year). It corresponds to a linear velocity of 1.4 mm/year of the upper part of this block. Displacements and tilts in other directions can be regarded as insignificant.

Block B moves horizontally at a rate of approx. 1.1 mm/year in a direction at a slight angle to the slope and at the same time subsides at an average rate of 0.5 mm/year. There are no significant tilts of the block B.

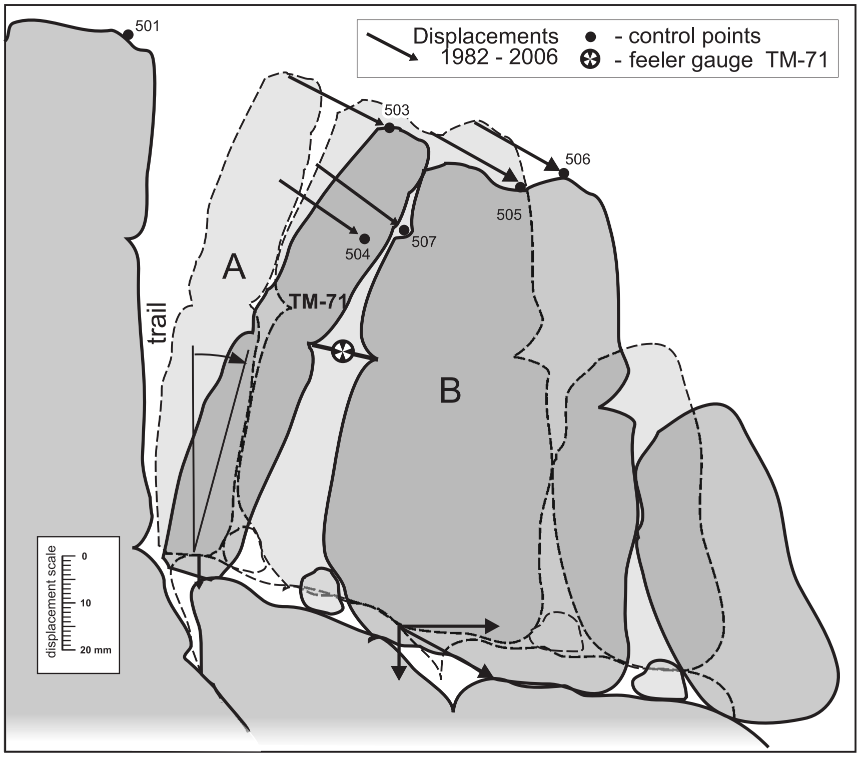

Fig. 6 - Schematic visualization of the rigid body movements of the blocks A and B

5. AN ATTEMPT TO INTERPRET THE PHENOMENON OF ROCK BLOCKS DEFORMATION IN THE "PIEKIELKO"

Unbiased interpretation of the rock block deformation phenomena from relatively short monitoring period (24 years) and its limited scope is very difficult. It arises from the fact that the present qualitative picture of this object of inanimate nature is the consequence of changes that started probably in the Cretaceous period (about 120 million years ago). Rate of these changes could have varied in different geological periods and their real causes are difficult to determine unambiguously. It is a fact that results of deformation measurements have confirmed present-day mobility of this object.

This attempt to interpret the phenomenon is based on the results of model research carried out by Kostak (1991). It had been realised in laboratory conditions using transparent tube, rectangular in section, and filled with plastic mass. Identical blocks were placed on its surface. The blocks were made of material that allowed keeping them on the plastic surface. Initial state of this experiment is shown in Fig 7a. Next the plastic mass was released through a hole in the bottom of the tube and the behaviour of individual blocks in time was filmed. Fig. 7b, 7c and 7d show characteristic positions of individual blocks registered at moments 1, 2 and 3.

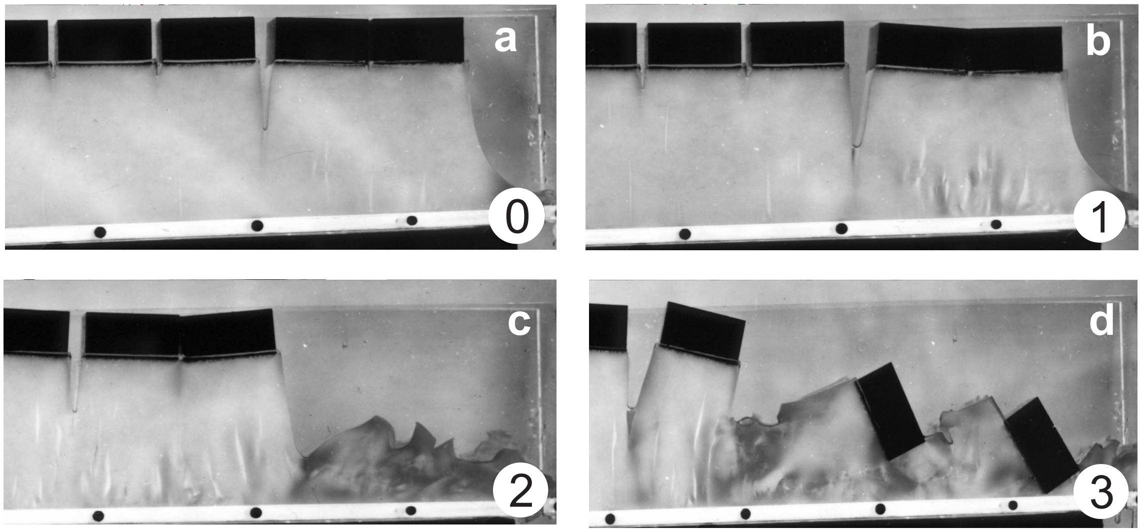

Fig. 7 - Model research of rock blocks deformations carried out by Kostak (1991)

Picture of these changes may be, with rough approximation, related to behaviour, in space and time, of bocks A and B in the “Piekielko”. It should be stated that the analysed blocks are located in the edge zone of the Szczeliniec Massif and situated on unstable marlstones. During heavy rainfalls their plasticity additionally increases and the process of suffosion causes fragmentation and washing of material from bed. This process is observed at a small distance from “Piekielko” – on the slope in several sources of the river Posna. In the end sliding of the rock blocks down the slope happens. Various forms of this can be observed in the ground below the analysed object.

6. CONCLUSIONS

The presented discussion concerning monitoring and analysis of rock block deformations in the “Piekielko” has demonstrated instability of this edge zone of the Szczeliniec Massif. Translation and rotation parameters of rock blocks have been calculated using simplified deformation model because of limitations related to number of measured research points on the analysed rock blocks.

Deformation parameters of rock blocks determined from results of geodetic and relative measurements indicate progressing process of the rock massif ‘s erosion. Geometry of motion of blocks obtained from measurements is consistent with model research carried out by Kostak. Rate of change of the measured points positions, at a level of millimetres per year, indicates relatively fast moving erosion process. Having in mind intensive tourist movement in region of the “Piekielko” rock cleft, monitoring of rock blocks deformations should be carried out continuously and the control-measurement system extended with additional control points and instruments for relative measurements. Development of research network with additional points will allow determining deformation parameters using models that include homogenous strains of blocks (dilatations, elongations, shears).

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