

## Application of Difference Methods in Surveying

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**Abstract** The paper reviews applications of difference-making methods used in GPS surveying and light-beam range finder measurement, and analyzes the advantages brought by difference-making, such as elimination of many kinds of errors that stems from both instruments and observations, and points out that the purpose of making difference is to eliminate both known and unknown common errors and thus improve measurement accuracy. Then by analogy the paper proposes Real-time Automatic Polar Coordinates Difference (RAPCD) method in deformation monitoring, which concentrates on three applications of the difference method in skew distance correction, earth-atmosphere difference correction, and azimuth correction, and the author presents relevant formulae for actual computations. And the method has been successfully applied in practical work, for example, the author has developed the software named “Automatic Deformation Monitoring System---ADMS” and has successfully applied it in a lot of engineering projects such as the Automatic Deformation

Monitoring of Guangzhou Subway, the Deformation Monitoring of the Long River Bridge of Yiling, and the Deformation Monitoring of the Yanzhou Reservoir Dam.

**Key words:** difference method; GPS surveying; range finder measurement; RAPCD method

## 1. Introduction

Errors occur when surveying instruments are used because of their own components and external environment. Some of the errors can be fixed. However, limited by current technology and knowledge, some errors can not or impossibly be determined, for example, internal phase shift of range finder and delay of the GPS signal caused by ionosphere or troposphere. One important characteristic of the errors whose value cannot be fixed is that they are stochastic. However, within a short period, they can be viewed as constants, so through surveying instruments with modern technology and by some means, they can be eliminated or reduced with their values unknown.

From the above idea does the difference method come into being whose purpose is to make difference between two values of two targets observed at one observation station, or two values of one target observed at two observation stations, or two values of one target at one observation station. By making difference the aim can be attained that known or unknown common errors can be eliminated and thus the accuracy of surveying results are improved.

## 2. Application of Difference Methods in GPS Surveying

GPS is a satellite positioning system with high accuracy. Difference methods are applied in GPS positioning. The technique of GPS positioning by making difference to be described in this section is that range correction from benchmark station to satellite is computed from a GPS receiver fixed on the benchmark station whose precise coordinates are known and transmitted in real-time so that users could make corrections to their observations with the range correction they have received as they are making GPS observations and hence improve the accuracy of positioning.

During the whole process of GPS positioning, there exist three parts of errors. The first part of errors is those every user receiver have in common, for example, satellite clock error, ephemerides error, ionosphere error and troposphere error; the second is signal propagation delay error that can't be determined by user's surveying or correction models; the third is those inherent in every user's receiver, for example, internal noises, channel delay, multi-path effect. By making difference, the first part of errors can be totally eliminated, and most errors of the second part can be eliminated (depending on the range from the benchmark station to the user receiver), while the third part can not be eliminated.

According to the kind of information transmitted by the benchmark station, difference of single GPS station can be divided into three types: position difference, pseudo-range difference and carrier phase difference, which share the same principles. The difference lies in the information transmitted by the benchmark station and as a result the positioning accuracy by making difference.

Currently pseudo-range difference is widely used and is applied in nearly all commercial GPS receivers. At the benchmark station, through observing all visible satellites, the geocentric coordinates  $(X^j, Y^j, Z^j)$  of each satellite can be computed from the ephemerides files collected, and then along with the known precise coordinates  $(X_0, Y_0, Z_0)$  of the benchmark

station, the real range  $R^j$  from each satellite to the benchmark station can be computed following equation(1):

$$R^j = \sqrt{(X^j - X_0)^2 + (Y^j - Y_0)^2 + (Z^j - Z_0)^2} \quad (1)$$

The value observed by receivers at the benchmark station is pseudo-range  $\rho_0^j$ , which is different from the real range. Hence the pseudo-range correction can be acquired as follows:

$$\Delta\rho^j = R^j - \rho_0^j \quad (2)$$

And the change rate of the pseudo-range correction is as follows:

$$\Delta\dot{\rho}^j = \frac{\Delta\rho^j}{\Delta t} \quad (3)$$

Then the benchmark station transmit  $\Delta\rho^j$  and  $\Delta\dot{\rho}^j$  to the user station. Adding the pseudo-range correction to the measured pseudo-range we can acquire the corrected pseudo-range:

$$\rho_p^j(t) = \rho^j(t) + \Delta\rho^j(t) + \Delta\dot{\rho}^j(t - t_0) \quad (4)$$

Based on the corrected pseudo-range  $\rho_p^j$ , if 4 satellites are observed then the coordinates  $(X_p, Y_p, Z_p)$  of the user station can be computed:

$$\rho_p^j = \sqrt{(X^j - X_p)^2 + (Y^j - Y_p)^2 + (Z^j - Z_p)^2} + C \cdot \delta t + v \quad (5)$$

with clock error  $\delta t$  and receiver noise  $v$ .

The first advantage of pseudo-range difference is that there is no coordinates conversion error because the computation of the pseudo-range correction is based on the coordinates system WGS-84 and need not converting into local coordinates. The second advantage is that positioning can be finished if the user receiver has observed 4 satellites. The third advantage is that the corrections this method provides guarantee continuous precise positioning within intervals when no corrections are available, which meets the RTCMSC-104 standard of the International Marine Radio Commission.

### 3. Application of Difference Method in Light-Beam Range Finder Measurements

An additional phase shift  $\delta$  is produced by internal electronic circuits of the range finder when ranging signals are transmitted. The phase difference measured by the phase measuring system contains a phase shift  $\Phi = \omega \cdot t_{2D}$  besides  $\delta$ :

$$\Phi = \omega \cdot t_{2D} + \delta \quad (6)$$

In equation (6),  $\omega \cdot t_{2D}$  is the phase change caused by the signal on the measured distance to and fro.

The value of  $\delta$  is influenced by not only the structure of electronic circuit but also stability of components, environment and instrument stability during start. All these contribute to an undeterminable zero point of phase which is the so-called “zero drift” problem. This problem cannot be solved by adding constant correction to the measurement and can only be solved in other ways.

In order to solve the above problem, a surveying method called “internal and external light path difference” is applied through setting an internal light path prism in the range finder as shown in Figure 1. The internal light path system consists of a little reflective prism and a light-conductive tube  $c$  and the little prism can be located at position  $a$  or  $b$ .

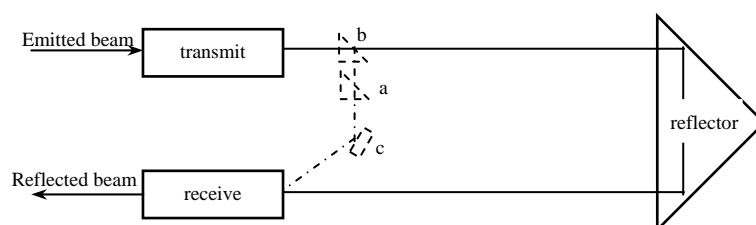


Figure 1: Principle of difference surveying of the internal and external light path

When the little prism is located at position a, the shoot beam will reach the prism station. It is called external light path measurement and can be formulated as follows:

$$\Phi_{\text{ext.}} = \omega \cdot t_{2D} + \delta \quad (7)$$

When the little prism is located at position b, the shoot beam will not reach the prism station but be reflected by the little prism to the incident point of the light conductive tube (the conductive tube is a piece of optical fiber) and then reach the receiving system through the light conductive tube. It is called internal light path measurement. The result can be formulated as follows:

$$\Phi_{\text{int.}} = \omega \cdot t_g + \delta \quad (8)$$

In equation (8),  $\omega \cdot t_g$  stands for the phase change produced by the light beam as it passes the little prism and one end of the light conductive tube.

The final result is the difference between the external light path and the internal light path:

$$\Phi = \Phi_{\text{ext.}} - \Phi_{\text{int.}} = \omega \cdot t_{2D} - \omega \cdot t_g = \omega \cdot (t_{2D} - t_g) \quad (9)$$

As for an instrument,  $\omega \cdot t_g$  is a fixed value and can be determined with instrument constants and is usually pre-determined in design with the mechanical structure of the instrument.

From the above we can see that the application of the internal and external light path difference method can eliminate the internal phase shift of the range finder very well and so the problem of zero drift is solved.

#### **4. Application of Real-time Automatic Polar Coordinates Difference (RAPCD) in Deformation Monitoring**

The difference methods can also be applied in deformation monitoring. During the process of deformation monitoring with a total station, there are a lot of errors such as vertical and horizontal deflection of the atmosphere, temperature and air pressure, and internal errors of instruments. It is extremely hard to evaluate the above errors. However, in order to reduce or eliminate the errors to improve the accuracy, the difference method can be applied.

If the difference method in the deformation monitoring with a total station, it is necessary for the total station to take proper time to complete all points in one period. So we can use automatic total station (for example, the TCA series total station produced by Leica) to build real-time automatic polar coordinates difference system to make deformation monitoring. Here the Real-time Automatic Polar Coordinates Difference (RAPCD) method proposed by the author will be presented.

##### **4.1 Difference Correction of Skew Distance**

The influence of the change of atmosphere conditions on range measurement must be taken into account in the polar coordinates deformation monitoring system. Generally speaking, in order to acquire the refractive correction of the atmosphere for distance, weather factors of the atmosphere need to be measured. Therefore, for the aim of automatization of deformation monitoring, digital thermometer and barometer are appended in some systems.

Usually there is a datum net used to detect the stability of datum points in the deformation monitoring network. With the help of information from the datum net, the atmosphere refractive correction for the skew distance can be made in real-time without measurement of weather factors and with a simplified system configuration.

The distance between the monitoring station and the datum point can be considered constant as they both are built in stable rock. Suppose the distance from the monitoring station to some datum point is  $d_J^0$ , and then during deformation monitoring, the skew distance is measured to be  $d_J'$ . The cause of the difference between the two values can be considered change of meteorological conditions. So following equation (10) the proportional factor  $\Delta d$  for the meteorological correction can be obtained:

$$\Delta d = \frac{d_J' - d_J^0}{d_J'} \quad (10)$$

In reality, an average of a lot of proportional factors  $\Delta d$  from many datum points for the meteorological correction is utilized for the meteorological difference correction for distance measurements to guarantee the reliability and accuracy of the proportional factor  $\Delta d$  of the meteorological correction for distance measurements.

If the skew distance of some deformation point is measured to be  $d_p'$  at the same moment, then the real skew distance corrected by meteorological difference can be formulated as follows:

$$d_p = d_p' - \Delta d \cdot d_p' \quad (11)$$

#### 4.2 Correction for Earth-Atmosphere Difference

To determine the three-dimension coordinates of the deformation points, the influence of the earth-atmosphere difference on height difference must be taken into consideration. As precise leveling has always been made on the datum net, the height difference  $\Delta h^0$  between the datum point and the monitoring point is known. At some moment suppose the observed trigonometrical height difference  $h_J$  between the monitoring station and the datum point is:

$$h_J = d_J \cdot \sin \alpha + i_h - a_h \quad (12)$$

in which  $\alpha$  being vertical angle,  $i_h$  being instrument height and  $a_h$  prism height,

and then according to the following formula the correction factor  $c$  for the earth-atmosphere difference can be computed:

$$c = \frac{\Delta h^0 - h_J}{d_p'^2 \cdot \cos^2 \alpha} \quad (13)$$

During the monitoring process of deformation points per period, as the time for measurement is short, the influence of the  $c$  on the datum point and the deformation point is same, so following equation (14) we can acquire the trigonometrical height difference  $\Delta h_p$  corrected with the earth-atmosphere difference between the deformation point and the monitoring point:

$$\Delta h_p = d_p \cdot \sin \alpha + c \cdot d_p'^2 \cdot \cos^2 \alpha + i_h - a_h \quad (14)$$

After the skew distances  $d_p$  and the height differences  $\Delta h_p$  between the monitoring station and every deformation points have been computed, following equation (15) we can get the

horizontal distance  $D_p$  between the monitoring station and the deformation points:

$$D_p = \sqrt{d_p^2 - \Delta h_p^2} \quad (15)$$

### 4.3 Difference Correction for Azimuth

It is difficult to guarantee the stability of instruments during long-term deformation monitoring. The influence of change of horizontal dial's zero on horizontal azimuth can not be ignored. In deformation monitoring, because all deformation variants are relative to the first period, thus we can take the azimuth  $H_{ZJ}^0$  observed of the datum point for the first time for datum azimuth. Hence the difference  $\Delta H_Z$  between azimuths measured relative to the datum point in other periods and the datum azimuth is as follows:

$$\Delta H_Z = H_{ZJ}' - H_{ZJ}^0 \quad (16)$$

The above difference is the result of the change of the horizontal dial's zero caused by instrument stability and horizontal refraction of the atmosphere. It has equal influence on measurements of deformation points, so the accurate azimuth  $H_{ZP}$  of deformation points can be acquired by adding the correction  $\Delta H_Z$  from datum points of the same period to the observed azimuth  $H_{ZP}'$  of the deformation point per period:

$$H_{ZP} = H_{ZP}' - \Delta H_Z \quad (17)$$

From the above difference corrections, the three dimension coordinates of every deformation point per period can be determined accurately in formulae of polar coordinates:

$$\left. \begin{aligned} X_p &= D_p \cdot \cos H_{ZP} + X^0 \\ Y_p &= D_p \cdot \sin H_{ZP} + Y^0 \\ Z_p &= \Delta h_p + Z^0 \end{aligned} \right\} \quad (18)$$

in which  $X^0, Y^0, Z^0$  are coordinates of the monitoring station.

Based on the principles of real-time polar coordinates difference, the author has developed the software named *Automatic Deformation Monitoring System---ADMS* and has it successfully applied in a lot of projects such as the Automatic Deformation Monitoring of Guangzhou Subway, Deformation Monitoring of the Long River Bridge of Yiling, Deformation Monitoring of the Yanzhou Reservoir Dam.

### 5. Conclusion and Prospect

In real work, there are many error control methods in which the difference method is very effective. In the difference method, though the value of an error can be unknown, yet it can be eliminated or reduced effectively just by analyzing properties of the error and thus the accuracy can be improved. In this paper, applications of traditional difference methods in GPS and the range finder are introduced, and then the automatic real-time difference of polar coordinates is proposed and described in detail.

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