

CONGRUENCE TESTS AND OUTLIER DETECTION IN DEFORMATION ANALYSIS WITH RESPECT TO OBSERVATION IMPRECISION

Ingo Neumann and Hansjörg Kutterer
Geodetic Institute
University of Hannover
Email: [neumann, kutterer]@gih.uni-hannover.de

Abstract: The quality of geodetic deformation analysis depends essentially on the adequate consideration of all uncertainties in the measurement and analysis process. In practice, only random variations (stochastics) are taken into account, although the non reducible systematics during the measurement process (observation imprecision) play a decisive role. In this study, the uncertainty budget of the observations is assumed to comprise both, stochasticity and observation imprecision, what leads to intervals or fuzzy intervals for their description. For this reason, it is necessary to extend the classical techniques of statistical hypothesis testing in a suitable way to check the accordance of the collected data with the assumptions met in the model. This requires one and multidimensional hypothesis tests with imprecise extensions for outlier detections, global tests and congruence tests in least-squares adjustment. It is shown, that the consideration of observation imprecision is an independent extension of the classical test approach. The new hypothesis tests are based on the intervals or fuzzy intervals of the observations. Numerical examples demonstrate an improved interpretation of the observations and model parameters, e.g., in epoch comparisons, as the main benefit.

1. Introduction

The aim of geodetic deformation analysis is the generation of meaningful motion and deformation models in order to quickly and specifically initiate constructional or geotechnical safety measures. The adequateness of the actions depends essentially on the quality of the measurements and analysis techniques and on the suitable consideration of all uncertainties. After calculating the uncertainty budget, the accordance of the collected data with the assumptions met in the model must be checked. Therefore, the well known classical test approaches are extended in order to take observation imprecision into account.

In this study, the uncertainty budget of the observations is assumed to comprise random variability and imprecision. Both uncertainties represent a special case of the so-called fuzzy randomness (see [3], [10] and [14]). We assume a precise stochastic component what is standard in geodetic data analysis. This component is superposed by imprecision due to unknown remaining systematic errors. Note that imprecise quantities are exclusively modeled in terms of fuzzy intervals.

A fuzzy interval \tilde{A} is uniquely defined by its membership function $m_{\tilde{A}}(x)$ over the set \mathbb{R} of real numbers with a membership degree between 0 and 1:

$$\tilde{A} := \{(x, m_{\tilde{A}}(x)) \mid x \in \mathbb{R}\} \quad \text{with} \quad m_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1] \quad (1)$$

The membership function of a fuzzy interval can be described by its left (L) and right (R) reference function (see also Fig. 1)

$$m_{\tilde{A}}(x) = \begin{cases} L\left(\frac{x_m - x - r}{c_l}\right), & x < x_m - r \\ 1, & x_m - r \leq x \leq x_m + r \\ R\left(\frac{x - x_m - r}{c_r}\right), & x > x_m + r \end{cases} \quad (2)$$

with x_m denoting the midpoint, r its radius, and c_l, c_r the spread parameters of the monotonously decreasing reference functions.

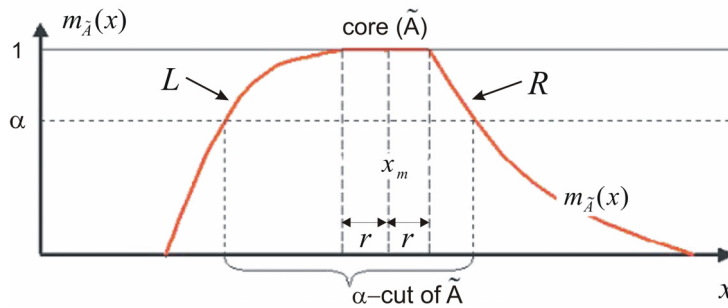


Figure 1: Fuzzy interval and its α -cut

A very important notion is the α -cut of a fuzzy interval, with $\alpha \in [0, 1]$:

$$\tilde{A}_\alpha := \{x \in X \mid m_{\tilde{A}}(x) \geq \alpha\}, \quad (3)$$

which is a classical interval for each α . The *core* of \tilde{A} is defined as $core(\tilde{A}) = \tilde{A}_1$. The integral over all α -cuts equals the membership function:

$$m_{\tilde{A}}(x) = \int_0^1 m_{\tilde{A}_\alpha}(x) d\alpha \quad (4)$$

Note that fuzzy intervals are a special case of the general concept of fuzzy sets according to Zadeh [15]. Hence, basic operations on fuzzy intervals are the *intersection* and the *complement*; they are defined through the following membership functions:

$$\text{Intersection:} \quad \tilde{C} = \tilde{A} \cap \tilde{B} \Leftrightarrow m_{\tilde{A} \cap \tilde{B}}(x) = \min(m_{\tilde{A}}(x), m_{\tilde{B}}(x)) \quad \forall x \in \mathbb{R} \quad (5_a)$$

$$\text{Complement:} \quad \tilde{C} = \tilde{A}^c \Leftrightarrow m_{\tilde{A}^c}(x) = 1 - m_{\tilde{A}}(x) \quad \forall x \in \mathbb{R} \quad (5_b)$$

There are also arithmetic rules which can be directly applied to fuzzy intervals. For further information on fuzzy-theory and interval mathematics, see [1], [3], [5]. Studies on fuzzy data analysis in the geodetic context are presented by [7], [9] and [13].

2. The purely stochastic case of hypothesis testing

Nowadays it is standard to use hypothesis testing to check the consistency of the collected data and the model. The well-known strategies of outlier detection and congruence check are briefly reviewed in the following two subsections since these two cases will be extended afterwards to take imprecision into account.

2.1. One- and multidimensional hypothesis testing for outlier detection

In geodesy, the model parameters \mathbf{x} are typically estimated from a large number of heterogeneous and redundant observations by means of a least-squares adjustment based on a Gauss-Markov model; see, e.g., [6]. For rank-deficient design matrices which are relevant in free geodetic networks it is common practice to derive the estimated parameters $\hat{\mathbf{x}}$ as a projection (pseudo-inverse solution):

$$\hat{\mathbf{x}} = \mathbf{x}_0 + (\mathbf{A}^T \mathbf{P}_{yy} \mathbf{A})^+ \mathbf{A}^T \mathbf{P}_{yy} \mathbf{y}, \quad (6)$$

with the $n \times u$ column-singular design matrix \mathbf{A} (rank deficiency d), the $n \times 1$ vector of approximate values \mathbf{x}_0 of the parameters \mathbf{x} , the $n \times n$ regular weight matrix $\mathbf{P}_{yy} = \mathbf{Q}_{yy}^{-1} = \sigma_0^2 \mathbf{C}_{yy}^{-1}$ (with the cofactor matrix \mathbf{Q}_{yy} and the variance-covariance matrix \mathbf{C}_{yy} of the observations, and the variance of the unit weight σ_0^2), the $n \times 1$ vector of reduced observations \mathbf{y} ("observed minus computed"). The pseudo-inverse matrix is denoted by the superscript $+$. The number of observations is n , and u is the number of parameters. The estimated residuals $\hat{\mathbf{v}}$ and their cofactor matrix $\mathbf{Q}_{\hat{\mathbf{v}}}$ are, e.g., obtained as

$$\hat{\mathbf{v}} = -\mathbf{Q}_{\hat{\mathbf{v}}} \mathbf{P}_{yy} \mathbf{y}, \quad \mathbf{Q}_{\hat{\mathbf{v}}} = \mathbf{Q}_{yy} - \mathbf{A}(\mathbf{A}^T \mathbf{P}_{yy} \mathbf{A})^+ \mathbf{A}^T. \quad (7)$$

The large number of observations in deformation analysis may cause outliers, which have to be detected and then removed from the data material. This requires both one- and multidimensional hypothesis tests. The main steps of hypothesis testing for outlier detection are briefly summarized in the following.

The calculation may start with the hypotheses for the estimated residuals $\hat{\mathbf{v}}_{sel} = \mathbf{B}^T \hat{\mathbf{v}}$ of the p observations to be tested simultaneously; see Table 1 for examples of matrix \mathbf{B} :

$$H_0 : E(\hat{\mathbf{v}}_{sel}) = \mathbf{0}, \quad H_A : E(\hat{\mathbf{v}}_{sel}) \neq \mathbf{0}. \quad (8)$$

A proper test statistic is given by the quadratic form

$$R = \mathbf{y}^T \mathbf{P}_{yy} \mathbf{Q}_{\hat{\mathbf{v}}} \mathbf{P}_{yy} \mathbf{B}(\mathbf{B}^T \mathbf{P}_{yy} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P}_{yy} \mathbf{Q}_{\hat{\mathbf{v}}} \mathbf{P}_{yy} \mathbf{y} / \sigma_0^2 \sim \chi^2(p, \delta), \quad (9)$$

that follows a central χ^2 -distribution (noncentrality parameter $\delta = 0$, with p degrees of freedom) with respect to the null hypothesis H_0 .

The test decision is based on the comparison of the actual value of R with the $(1 - \gamma)$ -fractile value of the χ^2 -distribution:

$$R > \chi_{1-\gamma}^2(p, 0) \Rightarrow \text{reject } H_0, \quad (10)$$

with the significance level of $1-\gamma$. If the value of the quadratic form exceeds the given $(1-\gamma)$ -fractile value of the χ^2 -distribution, the test is rejected and the p observations are considered as outliers. A test rejection may be caused by inadequate choices of stochastic and/or functional model components.

Alternative hypotheses H_A	Matrix \mathbf{B}
Outliers in observations l_i and l_k	$\mathbf{B}^T = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 \end{bmatrix}$
Incorrect GPS-baseline	$\mathbf{B}^T = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 0 & 1 & 0 & \dots & 0 \end{bmatrix}$

Table 1: Examples for the matrix \mathbf{B} [12]

In the one dimensional case the strategy leads to the well known test of standardized residuals [2]; it can also be used for significance tests of estimated parameters [6].

2.2. Congruence tests

The reliability of detecting the object movements depends essentially on the realization of a stable monitoring network around the interesting object. For this reason, the stability of the network points has to be tested with congruence tests as to possible differences between the two epochs. These procedures are described in detail in [12], the basic aspects of the formalism for the comparison of two epochs are summarized in the following.

For a straightforward computation, we assume in both epochs the same configuration, the same approximate values for the parameter vector and the same selection of datum points. The calculation starts with the definition of the estimated parameter vectors $\hat{\mathbf{x}}_i$ of the epochs i :

$$\hat{\mathbf{x}}_i = \mathbf{x}_0 + (\mathbf{A}_i^T \mathbf{P}_i \mathbf{A}_i)^+ \mathbf{A}_i^T \mathbf{P}_i \mathbf{y}_i \quad \text{with } i = 1, 2 \quad (11)$$

The difference vector \mathbf{d} between the parameter vectors of two epochs is introduced as:

$$\mathbf{d} = \hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2 = \left[(\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1)^+ \mathbf{A}_1^T \mathbf{P}_1 \quad -(\mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2)^+ \mathbf{A}_2^T \mathbf{P}_2 \right] \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \mathbf{Q}_{\mathbf{xd}} \bar{\mathbf{y}}, \quad (12)$$

with $\mathbf{Q}_{\mathbf{xd}} = \left[(\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1)^+ \mathbf{A}_1^T \mathbf{P}_1 \quad -(\mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2)^+ \mathbf{A}_2^T \mathbf{P}_2 \right]$ and $\bar{\mathbf{y}} = [\mathbf{y}_1 \quad \mathbf{y}_2]^T$. The hypotheses considered are:

$$H_0 : E(\mathbf{d}) = \mathbf{0}, \quad H_A : E(\mathbf{d}) \neq \mathbf{0}. \quad (13)$$

Under the null hypothesis, the parameters are assumed to be equal in both epochs. The cofactor matrices of the parameters in both epochs are given by $\mathbf{Q}_{\hat{\mathbf{x}}_1} = (\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1)^+$ and $\mathbf{Q}_{\hat{\mathbf{x}}_2} = (\mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2)^+$. The cofactor matrix $\mathbf{Q}_{\mathbf{dd}}$ of the difference vector \mathbf{d} is obtained as:

$$\mathbf{Q}_{dd} = \mathbf{Q}_{\hat{x}_1\hat{x}_1} + \mathbf{Q}_{\hat{x}_2\hat{x}_2} = (\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1)^+ + (\mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2)^+ \quad (14)$$

Then, the test statistics of the congruence test is a quadratic form:

$$K = \mathbf{d}^T \mathbf{Q}_{dd}^+ \mathbf{d} / \sigma_0^2 = \bar{\mathbf{y}}^T \mathbf{Q}_{xd}^T \mathbf{Q}_{dd}^+ \mathbf{Q}_{xd} \bar{\mathbf{y}} / \sigma_0^2 \sim \chi^2(h-d, 0) \text{ under } H_0, \quad (15)$$

with h the number of tested parameters in one epoch. The comparison of the actual value of the quadratic form with the $(1-\gamma)$ -fractile value of the χ^2 -distribution leads to the test decision

$$K > \chi_{1-\gamma}^2(h-d, 0) \Rightarrow \text{reject } H_0. \quad (16)$$

If the null hypothesis is rejected, one can assume changes of the network between the two epochs with a significance level of $1-\gamma$.

3. Hypothesis testing with respect to observation imprecision

In order to take observation imprecision into account, the quadratic forms R and K from the Sections 3.1 and 3.2 have to be extended for imprecise vectors of reduced observations $\tilde{\mathbf{y}}$. The presented test strategy is the multi-dimensional extension of the work presented in [8] which is based on [14].

3.1. Using α -cut optimization for the construction of the imprecise test value

The imprecise vector of reduced observations $\tilde{\mathbf{y}}$ is obtained by means of a sensitivity analysis with respect to some influence parameters $\tilde{\mathbf{p}}$ for a sufficient number of α -cuts (intervals)

$$\mathbf{y}_m = \mathbf{f}(\mathbf{p}_m); \tilde{\mathbf{y}}_{r_\alpha} = |\mathbf{F}| \tilde{\mathbf{p}}_{r_\alpha}; \mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{p}} \rightarrow [\tilde{\mathbf{y}}_\alpha] = \mathbf{y}_m + \mathbf{F} \cdot [-\tilde{\mathbf{p}}_{r_\alpha}, \tilde{\mathbf{p}}_{r_\alpha}] \quad (17)$$

with the matrix of partial derivatives \mathbf{F} to linearize the system of equations, the midpoint of the imprecise influence parameters \mathbf{p}_m and the midpoint of the imprecise vector of reduced observations \mathbf{y}_m . The radius of an α -cut of the reduced observations is given by $\tilde{\mathbf{y}}_{r_\alpha}$ and the radius of an α -cut of the influence parameters is defined by $\tilde{\mathbf{p}}_{r_\alpha}$. The midpoints are the carriers of stochasticity and the spreads are the carriers of imprecision. For a more detailed description see [13].

The fuzzy evaluation of a quadratic form $\mathbf{y}^T \mathbf{C}_{yy}^{-1} \mathbf{y}$ with $\mathbf{y} \in \tilde{\mathbf{y}}$ or $\mathbf{y} \in \mathbf{y}_m + \int_0^1 \mathbf{F} \tilde{\mathbf{p}}_{r_\alpha} d\alpha$ is based on Zadeh's extension principle [15]. If the quadratic form fulfills the criteria

- convexity
- continuity onto mapping

the extensional principle can be equivalently replaced by the min-max operator of an optimization algorithm [10]. The properties above are satisfied in a least squares adjustment with convex fuzzy intervals for the influence parameters $\tilde{\mathbf{p}}$.

Due to the convex function we use a local Newton algorithm to minimize/maximize a quadratic function subject to bounds of the variables [4]. The imprecise test value $\tilde{T} = \tilde{\Omega}$ is

constructed based on the min-max values of the quadratic form for a sufficient number of α -cuts, obtained by the described optimization method. Figure 2 demonstrates this procedure.

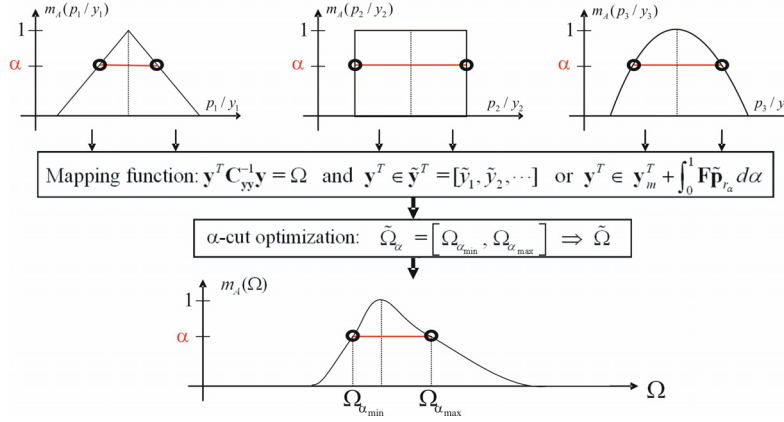


Figure 2: Construction of an imprecise test value $\tilde{T} = \tilde{\Omega}$ using α -cut optimization

3.2. Test decision based on the *card* criterion with respect to observation imprecision

The final test decision is based on the set-theoretical comparison of the imprecise test value (constructed using an α -cut optimization algorithm) with the region of acceptance \tilde{A} and the region of rejection \tilde{R} (see Fig. 3). For detailed information, concerning the test decision and the test criteria please refer to [8] and [11]. The hypotheses are defined by

$$T_m \sim \chi^2_{1-\gamma}(f, \delta); \quad \delta = \begin{cases} 0 & | H_0 \text{ the null hypothesis,} \\ \delta \neq 0 & | H_A \text{ the alternative hypothesis,} \end{cases} \quad (18)$$

with the noncentrality parameter δ and the significance level $1-\gamma$. The midpoint of the test value follows under the null hypothesis a central χ^2 -distribution with f degrees of freedom. The regions of acceptance \tilde{A} and rejection $\tilde{R} = \tilde{A}^c$ are defined as fuzzy intervals. The degree of the rejectability $\rho_R(\tilde{T})$ of the null hypothesis H_0 under the condition of \tilde{T} is computed based on the degree of agreement of the test value with the region of rejection $\gamma_{\tilde{R}}(\tilde{T})$ and the degree of disagreement of the test value with the region of acceptance $\delta_{\tilde{A}}(\tilde{T})$. We use the card criterion, because it allows a more suitable description of the degree of agreement between two fuzzy intervals. This leads to the equations given below (see Fig. 3):

$$\gamma_{\tilde{R}}(\tilde{T}) = \frac{\text{card}(\tilde{T} \cap \tilde{R})}{\text{card}(\tilde{T})} \quad \text{and} \quad \delta_{\tilde{A}}(\tilde{T}) = 1 - \frac{\text{card}(\tilde{T} \cap \tilde{A})}{\text{card}(\tilde{T})} \quad (19_a)$$

$$\rho_R(\tilde{T}) = \min(\gamma_{\tilde{R}}(\tilde{T}), \delta_{\tilde{A}}(\tilde{T})) \quad (19_b)$$

For the final test decision, the degree of rejectability $\rho_R(\tilde{T})$ of the null hypothesis has to be compared with a suitable critical value $\rho_{crit} \in [0, 1]$:

$$\rho_R(\tilde{T}) > \rho_{crit} \Rightarrow \text{reject } H_0 \quad (20)$$

The test is only rejected, if the test value agrees with the region of rejection and disagrees with the region of acceptance. This is in full accordance with the theoretical expectations, where observation imprecision is an additive term of uncertainty during the measurement process.

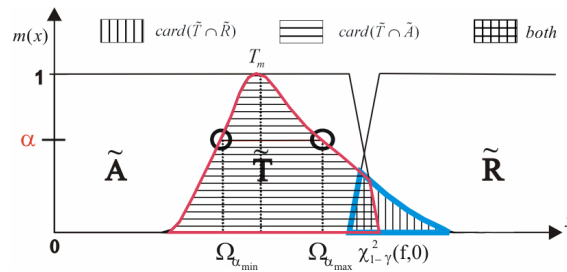


Figure 3: Comparison of the test value \tilde{T} with the regions of acceptance \tilde{A} and rejection \tilde{R} . The choice of ρ_{crit} depends on the particular application and must be based on expert knowledge. For outlier detection we propose to choose $\rho_{crit} \geq 0.5$ and for safety-relevant measures $\rho_{crit} \rightarrow 0$.

4. Test applications with respect to observation imprecision

In this section we show two numerical examples for the presented test strategy. The first example demonstrates an application of multiple hypothesis testing to outlier detection. The second example deals with a congruence test of a two dimensional geodetic network which was observed in two epochs. The test values are constructed at ten different α -cut levels. The membership function of the imprecise regions of acceptance \tilde{A} is equal to one until it reaches the fractile value $\chi^2_{0,95}(f, 0)$ and then decreases linearly to zero which is reached with the fractile value $\chi^2_{0,99}(f, 0)$. The influence parameters $\tilde{\mathbf{p}}$ of the sensitivity analysis are defined as triangular fuzzy intervals (linear reference functions).

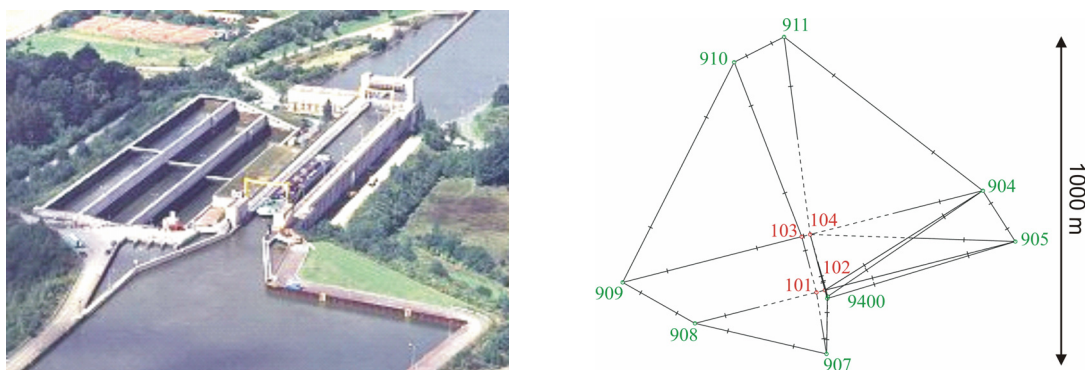


Figure 4: The lock Uelzen I and the geodetic monitoring network

We focus in our examples to a geodetic network called “lock Uelzen I”. The network is composed of eight control points around the lock and four points (101-104) on top of the lock (see Figure 4). Table 2 shows some typical orders of magnitude for the standard deviations σ and interval radii l_{r_α} (α -level of zero) of the observations.

	Distances	Zenith angles	Horizontal angles
$l_{r_\alpha} (\alpha=0)$	0,5 mm	0,5 mgon	0,1 mgon
σ	3 mm+ 2 ppm	1.5 mgon	0.5 mgon

Table 2: Typical interval radii and standard deviations of the observations

4.1. Outlier detection in the geodetic network

The first example shows a multiple outlier test due to an assumed centring error of the instrument, while measuring the horizontal angles at station 904. The construction of the test value \tilde{T}_{904} is based on the imprecise evaluation of the quadratic form in Eq. (9) with the described α -cut optimization method (see Section 3.1). In this example, the number of tested parameters is five ($p = 5$). Figure 5 shows the numerical test situation.

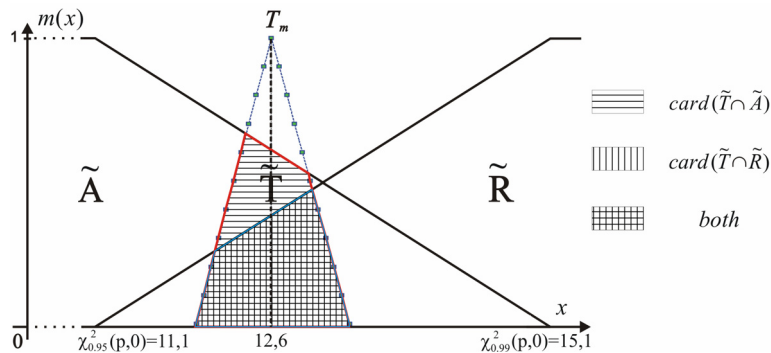


Figure 5: Multiple outlier test for horizontal angle measurements at station 904

The degree of rejectability $\rho_R(\tilde{T})$ of the null hypothesis H_0 is obtained as:

$$\gamma_{\tilde{R}}(\tilde{T}) = \frac{\text{card}(\tilde{T} \cap \tilde{R})}{\text{card}(\tilde{T})} = 0.60; \quad \delta_{\tilde{A}}(\tilde{T}) = 1 - \frac{\text{card}(\tilde{T} \cap \tilde{A})}{\text{card}(\tilde{T})} = 0.16 \rightarrow \rho_R(\tilde{T}) = 0.16 \quad (21)$$

The test decision, adapted from the hypotheses

$$T_m \sim \chi_{1-\gamma}^2(p, \delta) \quad \text{with} \quad H_0 : \delta = 0 \quad \text{and} \quad H_A : \delta \neq 0 \quad (22)$$

is obtained by:

$$\rho_R(\tilde{T}) = 0.16 < \rho_{crit} = 0,8 \Rightarrow \text{do not reject } H_0 \quad (23)$$

Obviously, in this example the observation imprecision is small. For this reason, the test value is tight and close to symmetric. The critical value ρ_{crit} is chosen to 0.8 because imprecision is an additive term of uncertainty what leads to a more reluctant rejection of the null hypothesis.

4.2. Congruence test for a geodetic monitoring network

The second example demonstrates an epoch comparison between the years 1999 and 2004. Both epochs are estimated within a partially constrained trace minimization with respect to the same six network points. The construction of the test value \tilde{T}_{con} is based on the imprecise evaluation of the quadratic form from Eq. (15) using the α -cut optimization method (see Section 3.1). Figure 6 shows the numerical test situation and the Tables 3 some specifications

about the two epochs and the network. Please note that the configurations in both epochs are different from each other, although it has not been shown formally here.

Specification	Epoch 1999	Epoch 2004
Observations n	317	144
Parameters u	60	39
Datum defects d	3	3

Table 3: Specifications about the monitoring network in the epochs 1999 and 2004

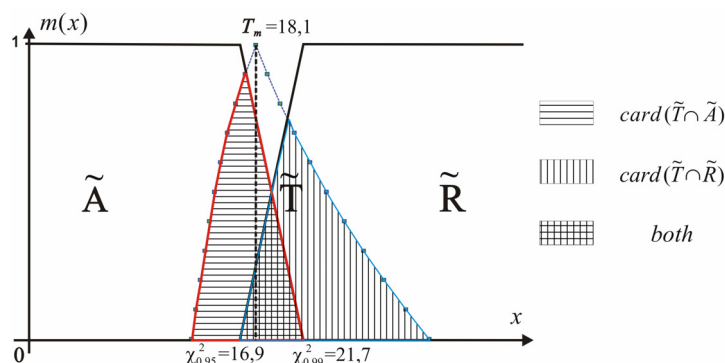


Figure 6: Test situation for a congruence test between the epochs 1999 and 2004

The degree of rejectability $\rho_R(\tilde{T})$ of the null hypothesis H_0 is obtained as:

$$\gamma_{\tilde{R}}(\tilde{T}) = \frac{\text{card}(\tilde{T} \cap \tilde{R})}{\text{card}(\tilde{T})} = 0,58; \quad \delta_{\tilde{A}}(\tilde{T}) = 1 - \frac{\text{card}(\tilde{T} \cap \tilde{A})}{\text{card}(\tilde{T})} = 0,52 \rightarrow \rho_R(\tilde{T}) = 0,52 \quad (24)$$

And the test decision, adapted from the hypotheses

$$T_m \sim \chi_{1-\gamma}^2(h-d, \delta) \quad \text{with} \quad H_0 : \delta = 0 \quad \text{and} \quad H_A : \delta \neq 0 \quad (25)$$

with $h = 12$ and $d = 3$ is obtained by:

$$\rho_R(\tilde{T}) = 0,52 < \rho_{crit} = 0,8 \Rightarrow \text{do not reject } H_0 \quad (26)$$

The strong imprecision of the test value in this example is caused by the bad effects of remaining systematics in epoch comparison of a monitoring network.

5. Conclusions

In this paper, we show a joint treatment of stochastics and observation imprecision in hypothesis testing. It is turned out that observation imprecision is an additive term of uncertainty which is caused by remaining systematics during the measurement process. For this reason the well known hypotheses tests are extended in order to take observation imprecision into account. This leads to a more reluctant rejection of the null hypothesis than in the pure stochastic case.

The presented test strategy allows to handle with imprecise variances and with Type I and Type II errors. Nevertheless, one important further generalization has to be mentioned. The imprecise test strategy has to be applied to F-distributions to consider a-posterior variances in the test situations. Furthermore, more real and simulated data sets should be under examination to make more refined decisions, e. g., about the critical value ρ_{crit} .

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